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Demand Behaviour for Weather Index Insurance Products in Regions Prone to Agricultural Droughts

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Abstract

Weather index insurance (WII) is a promising insurance scheme relevant for the agricultural sector, particularly in many low-income countries, including regions prone to agricultural droughts. WII policies might be more affordable to farmers, as the payouts in these insurance schemes are based on weather indices objectively determined for the specific agricultural regions, and therefore a costly individual loss assessment is not necessary. To successfully implement and scale up WII schemes, the development of transdisciplinary models properly addressing the complexity of relevant socio-natural processes and their interplay is necessary. We develop a stochastic model to simulate the demand for insurance policies in a drought-prone region under an assumption that weather and climate services, as tailored products for regions vulnerable to droughts, provide forecasts of the weather index on which the insurance scheme is based. Therefore, the simulated demand for the insurance policy might depend on the quality of the available weather index forecast. Presented modelling results suggest that both the income of individual producers and the dynamics of aggregate demand for insurance policies might be sensitive to the quality of the available weather index forecast. The developed modelling approaches can inform the design and implementation of WII schemes in drought-prone regions.

1 Introduction

With many substantial negative impacts on economy, society and ecosystems projected for moderate scenarios of climate change, and even more so for high-end scenarios [1–6], the problem of global warming calls for adaptation and mitigation actions informed by science [7]. Agriculture is considered the economic sector most vulnerable to climate change, and in many regions, weather- and climate-related risks for the agricultural sector are dramatic [8,9]. Crop insurance is an important mechanism of agricultural risk transfer because it redistributes the burden of the risk between the farmers and the insurance companies. But, in general, it is often expensive and unaffordable to many farmers, especially in low-income countries. One of the reasons for why crop insurance premiums are so high is the expenditures for loss assessment. In contraposition, in weather index insurance (WII)

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schemes the payouts are based on weather index objectively determined for the specific agricultural region, and therefore the individual loss assessment is not necessary. This paves the way to cheaper and more affordable agricultural insurance [10–12].

Among the 17 Sustainable Development Goals (SDGs), SDG 2 puts special emphasis on reducing hunger to zero [13]. We suggest that redistributing the risk of agricultural failures through crop WII insurances might support SDG 2 because the implementation of weather-based insurances indicates a high risk-reducing potential, therefore it increases the resilience of farming systems towards risks. Additionally, broad implementation of weather insurance schemes would support not only SDG 2 on zero hunger, but also SDG 10 on reducing inequalities and SDG 13 on climate action [13].

Assessments of historical impacts of hazardous events throughout the 20th century suggest that droughts have had the greatest negative impacts of all natural hazards [14–16]. The most affected population segment has been the rural poor, of which roughly 2.5 billion derive their livelihoods from agriculture [17]. In many regions of the world, droughts have huge impacts on agriculture, and, unsurprisingly, the agricultural drought (defined as the decreased availability of moisture for crops) enters the standard drought classification as one of few basic drought types [18]. Agricultural drought also means the loss of subsistence. A World Bank study shows that 1 percent growth in GDP from agriculture increases the expenditures of the three poorest deciles by at least 2.5 times as much as 1 percent growth from the rest of the economy [17]. Insurance from drought is one of the more promising types of WII to reduce the potential losses.

At the same time, WII is not a panacea [19], and its technical implementation is often very challenging [12, 20]. In particular, empirical studies often reveal low uptake rates not only for agricultural insurance in general [21], but also for existing pilot WII products in particular [12, 20].

We argue that, among many other factors, the strategies of individual producers in a region prone to agricultural droughts regarding the WII and their income might be substantially influenced by the availability and quality of the weather index forecasts for the region. The latter might be derived from seasonal weather forecasts. A weather index forecast is one possible product of weather/climate services tailored to the specific drought-prone region to inform and support the local adaptation measures [22–24]. While WII is very much related to short-term forecasts, the importance of its development is key for many developing countries in the world in which climate projections show that climate change might exacerbate droughts.

This paper contributes to the existing literature on modelling the relations between weather index forecasting and functioning of WII schemes [25–27] by developing the stochastic models of uptake of WII products [28].

The paper is organized as follows. Sec. 2 briefly describes a model of WII. In Sec. 3 we introduce a simple stochastic model connecting the actual weather index (on which the payouts of WII are based) to the forecasted weather index (that, generally, might be derived from seasonal weather forecasts and might represent one of weather/climate services tailored for the region under study). Sec. 4 describes strategies of individual producers with respect to buying the insurance policies, provides the framework for assessing producer's income and explores the possible role of adverse selection. Sec. 5 provides illustrative examples for the developed approach. In Sec. 6 simple models of the dynamics of aggregate demand for WII policies are introduced. Sec. 7 concludes.

2 Model of weather index insurance

The model of WII presented below is largely based on a model described in [29]. A key variable for our study is the weather index W. The weather index aggregates the weather information within the vegetation period. This might be a rather simple index, e.g. based on precipitation only, or a more complex construct additionally dependent on e.g. daily air temperature through thermal units or Growing Degree Days (GDD).[a] It should be stressed that in real-world applications W is a region- and crop-specific variable [29, 30].

We assume that low values of weather index W correspond to the situation of agricultural drought, while

[a] Growing Degree Days (GDD) is a measure of heat accumulation. GDD parametrize the impact of temperature on plant growth [29, 31].

high values of W are good for crops.

Let $W_{A,t}$ be the actual value of weather index at time t. In the context of our study, we assume for simplicity one harvest per year, and t enumerates the year. In the rest of the present section, the subscript t will be omitted, and the actual weather index will be denoted simply as W_A.

In general, W_A is derived from one or several weather or climate variables, like precipitation and temperature, that, in turn, are controlled by complex regional weather and climate dynamics [12]. However, for our purposes we make a simplification and model W_A as an uncorrelated random process with the lower and the upper bound,

$$W_{\min} \leq W_A \leq W_{\max}, \tag{1}$$

that can be described by a certain probability density function (PDF), $\pi(W_A)$. Illustrative examples in the present paper are provided for the case of uniform PDF:

$$\pi(W_A) = \begin{cases} 1/\Delta W, & \text{if} \quad W_{\min} < W_A < W_{\max}, \\ 0, & \text{otherwise}, \end{cases} \tag{2}$$

where a notation is introduced

$$\Delta W = W_{\max} - W_{\min}. \tag{3}$$

Crop yield y is supposed to depend on W_A and on an additive uncorrelated random noise ε with zero mean accounting for the basis risk

$$y = g(W_A) + \varepsilon. \tag{4}$$

For our theoretical analysis we will choose a simple functional dependence $g(W_A)$ steadily increasing with $g(W_A)$. Specifically, we assume a power law

$$y = W_A^{\beta}, \quad \beta = \text{const}, \quad 0 < \beta < 1. \tag{5}$$

With the PDF defined by Eq. (2), the average crop yield

$$\bar{y} = \int y(W)\pi(W)dW \tag{6}$$

is equal in this case to

$$\bar{y} = \frac{1}{\beta+1} \cdot \frac{W_{\max}^{\beta+1} - W_{\min}^{\beta+1}}{\Delta W}. \tag{7}$$

Following the concept of WII [12, 27, 29], the payout PO in the current year depends on the realization of W_A in that year and is equal to

$$PO(W_A) = \gamma \cdot \max(0, S - W_A), \tag{8}$$

where γ is a constant scaling factor called the tick size, and S is the strike level – the threshold at which the payout is triggered. Specifically, one might assume that

$$S = \xi \cdot W^*, \tag{9}$$

where ξ is a constant scaling factor ($0 < \xi < 1$) and W^* is the level of the weather index corresponding to the average crop yield (Eq. (6)),

$$W^* = g^{-1}(\bar{y}). \tag{10}$$

If the crop yield is parameterized by Eq. (5), the strike level can be expressed as

$$S = \xi \bar{y}^{1/\beta}. \tag{11}$$

The actuarially fair insurance premium P should be equal to average payout:

$$P = \overline{PO(W)} = \int PO(W)\pi(W)dW. \tag{12}$$

In our case,

$$P = \frac{\gamma}{2} \cdot \frac{(S - W_{\min})^2}{\Delta W}. \tag{13}$$

In the illustrative simulations provided in this paper, the following (non-dimensional) values of model parameters are used: $W_{\min} = 0.3$, $W_{\max} = 1.5$, $\beta = 1/3$, $\gamma = 2.0$, $\xi = 0.86$. This yields $W^* = 0.85$, $S = 0.73$, $P = 0.15$.

3 Model of forecast-based weather index

We now assume that a seasonal weather forecast is available for the region under study, and that, in general, a forecast of the weather index for the coming season might be made on the basis of that seasonal weather forecast (this might be one of the weather/climate services tailored for the region).

Let $W_{\mathrm{F},t}$ be the value of weather index at time t derived from seasonal weather forecast for period t. In general, estimating $W_{\mathrm{F},t}$ might be a challenging task [12]. What interests us now, however, is the relation between the forecast-based weather index $W_{\mathrm{F},t}$ and the actual weather index $W_{\mathrm{A},t}$. Obviously, the quality of forecasting the weather index should depend on the quality of the available seasonal weather forecast. In general case, we can define a relation between $W_{\mathrm{F},t}$ and $W_{\mathrm{A},t}$ by a joint PDF $\pi\left(W_{\mathrm{A}}, W_{\mathrm{F}}\right)$. For the purposes of our study we consider an illustrative example of a simple linear stochastic model

$$W_{\mathrm{F},t} - \overline{W} = k\left(W_{\mathrm{A},t} - \overline{W}\right) + \Delta + \varepsilon_t, \tag{14}$$

where the slope $k = \mathrm{const}$, $\overline{W}$ is the average weather index value equal in case of the model (2) to

$$\overline{W} = \frac{W_{\min} + W_{\max}}{2}, \tag{15}$$

Δ is the constant systematic error and ε_t is a random error. Our assumption regarding ε_t is that it is an uncorrelated random process with a uniform PDF

$$\pi(\varepsilon_t) = \begin{cases} 1/(2B), & \text{if} \quad -B < \varepsilon_t < B, \\ 0, & \text{otherwise}, \end{cases} \tag{16}$$

where $\pm B$ are the bounds of the random error.

In what follows below we will consider cases of positive, zero, and negative slope k.

Eq. (14) can be conveniently rewritten to the form

$$W_{\mathrm{F},t} = kW_{\mathrm{A},t} + b + \varepsilon_t, \tag{17}$$

where

$$b = (1-k)\overline{W} + \Delta. \tag{18}$$

Obviously,

$$kW_{\mathrm{A}} + b - B \leq W_{\mathrm{F}} \leq kW_{\mathrm{A}} + b + B \tag{19}$$

for the PDF given by Eq. (16).

In part of theoretical analysis below we will consider the limiting case of Eqs. (14), (17) with vanishing random error ($\varepsilon_t = 0$).

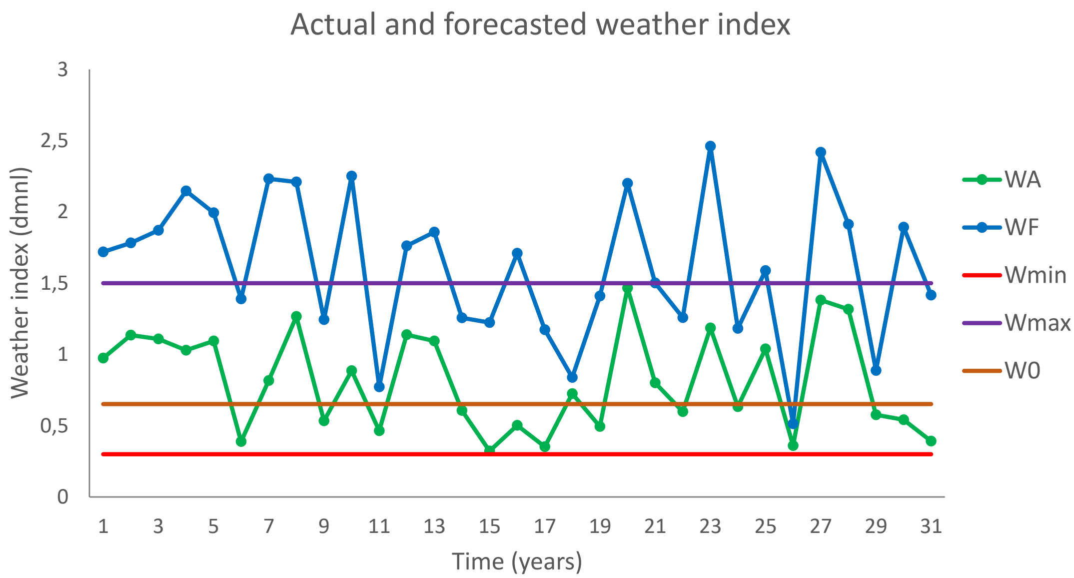

Fig. 1 The dynamics of actual weather index $W_{A,t}$ (green line) defined by Eq. (2) as an uncorrelated random process with uniform distribution (its upper bound W_{max} (lower bound W_{min}) is depicted by horizontal violet (red) line). The forecasted weather index $W_{F,t}$ (blue line) is related to the actual weather index $W_{A,t}$ through a statistical model (14) with a constant systematic error and an uncorrelated random error. The critical level of the weather index W_0 (depicted by the horizontal brown line) is defined by Eq. (22).

The statistical relation (14) of forecasted to actual weather index is illustrated in Figure 1 for the case $k = 1.0$, $\Delta = 0.8$, $B = 0.7$. For this numerical example, a rather large positive systematic error Δ is chosen, and an uncorrelated random error also has a significant standard deviation. One of the possible realizations of random dynamics of actual weather index $W_{A,t}$ is depicted by a green line in Figure 1. As defined by Eq. (2), the actual weather index is simulated as an uncorrelated random process with uniform distribution. Its upper and lower bounds (W_{max} and W_{min}) are depicted by horizontal violet and red line respectively. The forecasted weather index $W_{A,t}$ is related to the actual weather index $W_{A,t}$ through a statistical model (14), one of the possible realizations is depicted by a blue line. The critical level of the weather index defined by Eq. (22) (see Sec. 4 below) is also depicted in Figure 1 (horizontal brown line). In fact, the positive systematic error Δ chosen for the example visualised in Figure 1 is so big that a substantial fraction of realisations of $W_{F,t}$ exceeds the upper bound W_{max}. In what follows below, we assume that if the forecasted values go outside the bounds $W_{min} \leq W_F \leq W_{max}$, the forecast is adjusted to the closest bound (in case of Figure 1, this would mean adjusting $W_{F,t} \equiv W_{max}$ when the value of $W_{F,t}$ provided by Eq. (14) exceeds W_{max}).

4 Possible strategies of individual producers

Below we explore different strategies of individual producers who have to decide on whether to buy an insurance policy for the coming year or not.

Suppose, firstly, that the weather index forecast is either unavailable to the producer or is ignored, and the producer always buys the insurance policy.

Let I_t be the producer's income in year t. Under this strategy,

$$I_t = y(W_{A,t}) + PO(W_{A,t}) - P \quad \text{for all} \quad t. \tag{20}$$

Consider the average income of the producer $\bar{I} = \langle I_t \rangle$. Under assumptions of zero-mean basis risk (Eq. (4)) and the actuarially fair premium (Eq. (12)), average income will be simply equal to the average yield,

$$\bar{I} = \bar{y}, \tag{21}$$

and will be the same as if the producer were never buying the insurance.

We now assume that the producer makes their decision on the basis of the weather index forecast, while the insurer does not use the forecast in their decision making and still sets the insurance premium (12) solely on the basis of unconditional PDF of the actual weather index. As discussed in detail in [27], this might lead to the situation of intertemporal adverse selection, where the producer tends to purchase the insurance policy only when the coming year is forecasted to be 'bad', but not for 'good' years. However, as will be seen from illustrative examples provided below, this adverse selection would actually be beneficial for producers only when the quality of forecast is good enough, but, on the contrary, might be highly unprofitable otherwise.

We define the critical level of the weather index, W_0, as the level for which the payout is equal to the premium. If one would know in advance with certainty that $W_A < W_0$, it is profitable to buy in advance the insurance policy for the coming season. On the contrary, if $W_A > W_0$, it would be profitable not to buy the policy.

W_0 can be defined from the equation

$$PO(W_0) = P. \tag{22}$$

Substituting Eq. (13) into Eq. (22), we get

$$W_0 = \frac{2S \cdot W_{\max} - S^2 - W_{\min}^2}{2\Delta W}. \tag{23}$$

Numerically, for the values of model parameters specified in Section 2, $W_0 = 0.65$.

In real world, no forecast is perfect. So we assume that the producer is making their decision regarding the insurance policy on the basis of the forecasted value of the weather index $W_{F,t}$. On the contrary, the producer's actual income will depend on the actual value of the weather index $W_{A,t}$ that, in general, will deviate from its forecasted value (Section 3):

$$I_t = \begin{cases} y(W_{A,t}) + PO(W_{A,t}) - P, & \text{if} \quad W_{F,t} < W_0, \\ y(W_{A,t}), & \text{if} \quad W_{F,t} > W_0. \end{cases} \tag{24}$$

In an illustrative example, we will assume that the relation of the forecasted value to the actual value is given by the statistical model (14). However, we first start with a graphical interpretation of the general case of an adverse selection process.

The situation of adverse selection for which profitability depends on the forecast quality is illustrated with a phase diagram in the axes (W_A, W_F) in Figure 2a. Areas (i) and (ii) correspond to the case of 'objectively bad' years, when $W_A < W_0$ and buying the insurance policy would be profitable. On the contrary, areas (iii) and (iv) correspond to 'objectively good' years ($W_A > W_0$), and buying the insurance policy would be unprofitable. However, the producer is advised by the forecast to buy the insurance policy in case $W_F < W_0$ (areas (i) and (iii)) and is not advised to do this otherwise ($W_F > W_0$, areas (ii) and (iv)). Therefore, the producer is rightly advised to buy insurance policy in the area (i) and not to buy it in the area (iv). In area (iii) the producer gets the wrong advice to buy the insurance policy for the coming year that will actually be 'good', while in area (ii) they are wrongly advised not to buy the insurance policy for the coming 'bad' year.

In a hypothetical case of the perfect forecast $W_{F,t} \equiv W_{A,t}$ (green diagonal line in Figure 2b), the situation is always either in area (i) or in area (iv) that paves the way to 'perfectly efficient' adverse selection. However, in a realistic case of imperfect forecast, the joint PDF $\pi(W_A, W_F)$ will be positive in a certain area (schematically depicted by blue colour in Figure 2b) that, generally, can occupy (partially or fully) all of the four areas, (i) to (iv). As will be shown below, this might lead to the situation of unprofitable adverse selection.

Note, however, that not every imperfect forecast would lead to unprofitable adverse selection; nor should it even be necessarily worse in this respect than the perfect forecast. An example of imperfect forecast leading to

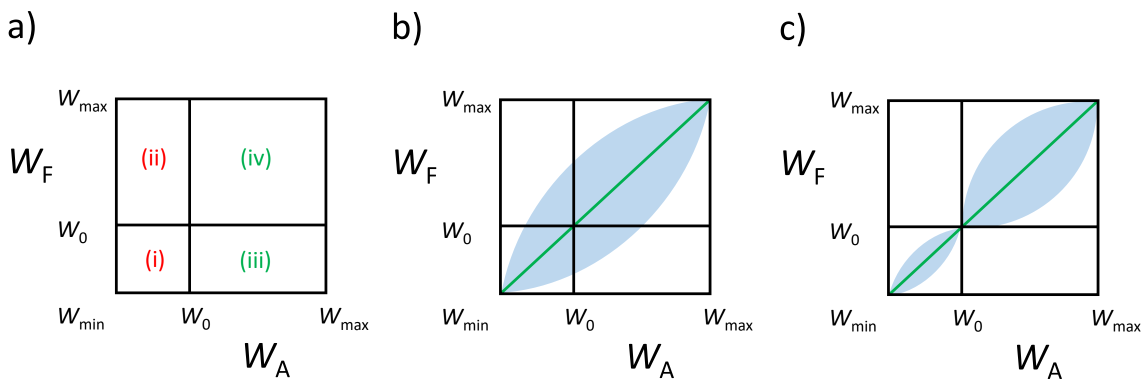

Fig. 2 (a) The phase diagram in axes (W_A, W_F) (actual weather index – forecasted weather index); the critical level of weather index W_0 breaks the phase plane into four areas, (i) to (iv). (b) Case of hypothetical perfect forecast (green diagonal line) and imperfect forecast (blue area). (c) Case of imperfect forecast (blue area) leading to as profitable adverse selection as the perfect forecast (green diagonal line).

the same decision making as the perfect one and therefore creating equally profitable adverse selection is shown in Figure 2c: here, again, the situation is always either in the area (i) or in the area (iv), and the imperfect forecast still gives perfectly efficient advice.

Based on Eq. (24), we can calculate the average income as

$$\begin{aligned}\bar{I} = &\int_{W_{min}}^{W_{max}} dW_A \int_{(W_F \le W_0)} dW_F\, \pi(W_A, W_F)\left[y(W_A) + PO(A) - P\right] \\ &+ \int_{W_{min}}^{W_{max}} dW_A \int_{(W_F > W_0)} dW_F\, \pi(W_A, W_F)\, y(W_A)\,.\end{aligned} \tag{25}$$

By rearranging the terms in the latter equation, we easily get

$$\bar{I} = \bar{y} + \overline{\Delta I}, \tag{26}$$

where

$$\overline{\Delta I} = \int_{W_{min}}^{W_{max}} dW_A \int_{(W_F \le W_0)} dW_F\, \pi(W_A, W_F)\left[PO(W_A) - P\right], \tag{27}$$

so the average income now is a sum of the average yield $\bar{y}$ and the surplus $\overline{\Delta I}$.

Integration by parts yields

$$\overline{\Delta I} = \gamma\left[\int_{W_{min}}^{S} F(w, W_0)dw - (S - W_0)F(W_0)\right], \tag{28}$$

where $F(w, W_0)$ is CDF (cumulative distribution function, or joint probability of $W_A < w$, $W_F < W_0$), and $F(W_0)$ is unconditional probability of $W_F < W_0$ ($F(W_0) = F(W_{max}, W_0)$).

As we will see below, dependent on CDF (that ultimately reflects the quality of the forecast), the surplus $\overline{\Delta I}$ can be positive, zero, or negative.

The dynamics of individual producer's income are shown in Figure 3 (brown line: 'always insurance' case; green line: hypothetical perfect forecast case; blue line: imperfect forecast case; effects related to the additive basis risk (Eq. (4)) are not superimposed on computed lines). Simulations shown in Figure 3 correspond to the same realizations of the dynamics of actual and forecasted weather index as shown in Figure 1. Producers supported in their decision-making by hypothetical perfect forecasts would benefit from 'perfectly efficient' adverse

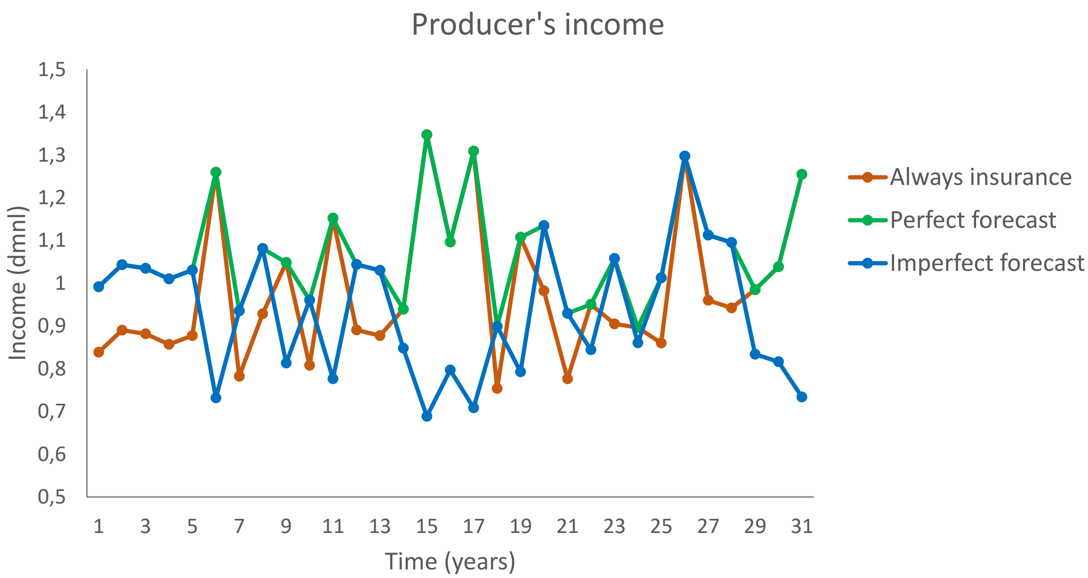

Fig. 3 The dynamics of individual producer's income (brown line: 'always insurance' case; green line: hypothetical perfect forecast case; blue line: imperfect forecast case) simulated for the same realizations of the dynamics of actual and forecasted weather index as shown in Figure 1.

selection. This makes the dynamics of producers' income under a perfect forecast scenario quite different from the imperfect forecast case (where the forecast quality is low and hence the probability of a wrong decision is high) and from the 'always insurance' case (where, by assumption, producers bear expenditures for insurance policies in all years, 'bad' and 'good').

5 Adverse selection and forecast quality: illustrative examples

We now explore the impact of forecast quality on the sign and the magnitude of the surplus ΔI for several illustrative examples.

Consider, firstly, the model (17) with zero random error,

$$W_{\mathrm{F}} = kW_{\mathrm{A}} + b, \tag{29}$$

where k and b can be positive, zero, or negative. This model will be depicted by a straight line on the plane $(W_{\mathrm{A}}, W_{\mathrm{F}})$. A full consideration of all possible cases of the model (29) is provided in Appendix A. Dependent on the values of k and b, the surplus $\overline{\Delta I}$ can be positive, zero, or negative.

In a particular case of Eq. (29) where the forecasted index deviates from its actual value only by the constant systematic error Δ,

$$W_{\mathrm{F}} = W_{\mathrm{A}} + \Delta, \tag{30}$$

the surplus $\overline{\Delta I}$ depends on Δ as follows (see Appendix B for the derivation):

$$\overline{\Delta I} = \begin{cases} (\gamma/(2\Delta W))\left[(W_0 - W_{\min})^2 - \Delta^2\right], & \text{if} -(S-W_0) < \Delta \le W_0 - W_{\min}, \\ (\gamma/\Delta W)\,(S - W_0)\,(W_{\max} - W_0 + \Delta)\,, & \text{if} -(W_{\max} - W_0) < \Delta \le -(S - W_0)\,, \\ 0, & \text{otherwise}. \end{cases} \tag{31}$$

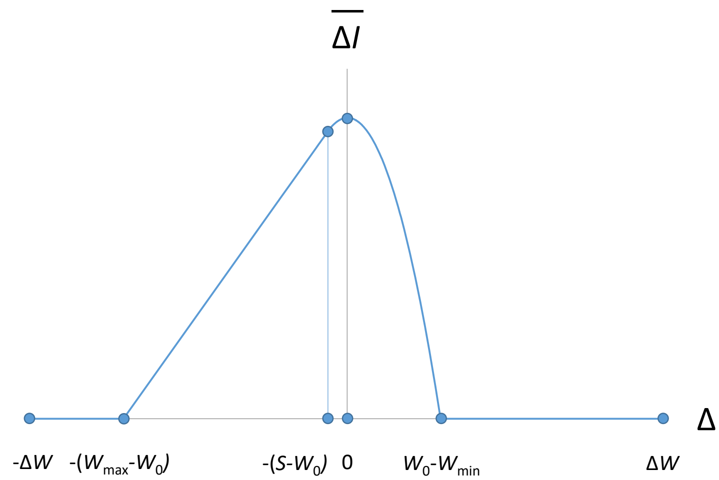

Fig. 4 The dependence of the surplus $\overline{\Delta I}$ to the producer's income on the systematic forecast error Δ in case of the model (30).

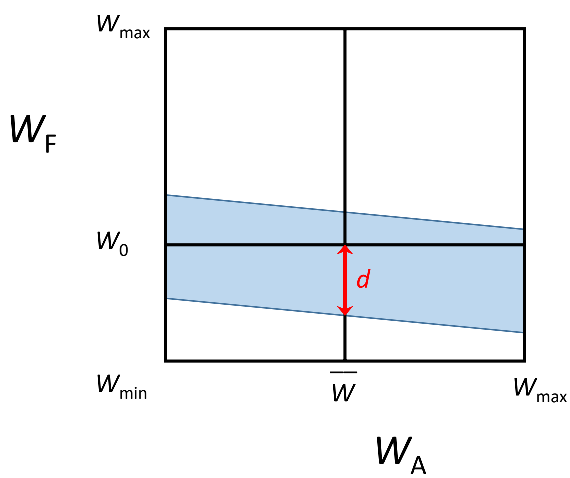

Fig. 5 The dependence of the forecasted weather index W_F on its actual value W_A provided by the stochastic model (14). Case of negative slope is shown ($k < 0$).

The first line of Eq. (31) reflects the quadratic dependence of $\overline{\Delta I}$ on Δ, while the second line – the linear one. In the connection point $\Delta = -(S - W_0)$ both $\overline{\Delta I}$ and its derivative over Δ are continuous. The maximum value of $\overline{\Delta I}(\Delta)$ is reached at $\Delta = 0$ (a hypothetical case of perfect forecast) and is equal to

$$\overline{\Delta I} = \frac{\gamma}{2\Delta W}(W_0 - W_{\min})^2 . \tag{32}$$

In this case, dependent on the value of Δ, the surplus can be either positive or zero, but is never negative (Figure 4).

We now turn to the case of the full stochastic model (14) with a non-zero random noise ε_t uniformly distributed within the band $-B < \varepsilon_t < B$ (Eq. (16)). We consider the model (14) when the systematic error Δ is fixed, while the absolute value and even the sign of the slope k may vary (in Figure 5, the case of $k < 0$ is shown). We introduce the average width of the band below the critical level W_0 of the weather index,

$$d = W_0 - \overline{W} + B - \Delta, \tag{33}$$

and require $d > 0$ (that means, the actual situation of the average weather index, $W_{\mathrm{A}} = \overline{W}$, can still be forecasted below the critical level W_0 with non-zero probability, Figure 5). Equivalently,

$$\Delta < W_0 - \overline{W} + B. \tag{34}$$

Additionally, we assume that $|k|$ is not big, to ensure that the lowest possible forecasted value $\tilde{W}_{\mathrm{F}}$ in case of $W_{\mathrm{A}} = W_{\min}$ is within the limits $W_{\min} \leq \tilde{W}_{\mathrm{F}} \leq W_0$ (it is easy to see that the same constraint applies to the the lowest possible forecasted value in case of $W_{\mathrm{A}} = W_{\max}$). An analysis presented in Appendix C shows that in this case the surplus is proportional to the slope k:

$$\overline{\Delta I} = k \frac{\gamma}{24B\Delta W} (S - W_{\min})^2 (3W_{\max} - W_{\min} - 2S). \tag{35}$$

The multiplier of k in Eq. (35) is always positive. So, the sign of the surplus $\overline{\Delta I}$ coincides with the sign of the slope k and can be positive, zero, or negative.

6 Model of the demand dynamics

Finally, we develop a simple model of the dynamics of the demand for WII policies. As discussed above, many different factors can affect WII market. In our modelling, we focus on the impacts of the quality of the weather index forecast.

In the hypothetical perfect forecast case, we will simply assume that the growth rate of the number of insured is constant and is equal to r:

$$\frac{dN}{dt} = rN. \tag{36}$$

Obviously, this would yield the exponential growth of the number of insured,

$$N(t) = N_0 \exp(rt). \tag{37}$$

In a general case of imperfect forecast, our assumption is that the growth rate is modulated by a factor dependent on whether the forecasted value of the weather index was related to its critical value W_0 in the same way as the actual value or not (case of areas (i) and (iv) in Figure 2a, as opposed to areas (ii) and (iii)):

$$\frac{dN}{dt} = \mathrm{sgn}(W_{\mathrm{A},t} - W_0)\mathrm{sgn}(W_{\mathrm{F},t} - W_0) rN, \tag{38}$$

where $\mathrm{sgn}(x)$ is a sign function equal to 1 if $x > 0$ and to -1 if $x < 0$. In the perfect forecast case, the model (38) is reduced to the previous exponential model (36). However, in the general case the solution of Eq. (38) is a random process that might look very differently from exponential growth (see Figure 6 for a numerical example).

7 Conclusions and outlook

Developing comprehensive models of WII schemes is important, as WII could have a great impact on making the access to insurances easier for farmers than it is today. As already mentioned, nowadays agricultural insurances are only affordable to rich farmers. This kind of insurance would also support the capability of farmers to have a back-up monetary solution in case of crop failure. Additionally, we assume that it would be easier to enhance the resilience of farming systems through WII when WII projects are supported by reliable forecasting tools.

The presented study contributes to theoretical modelling of certain dynamical features of WII project implementation. Specifically, it is focussed on modelling the strategies of producers in a drought-prone region regarding the WII policy purchase, and also on simulating the dynamics of aggregate demand for this kind of

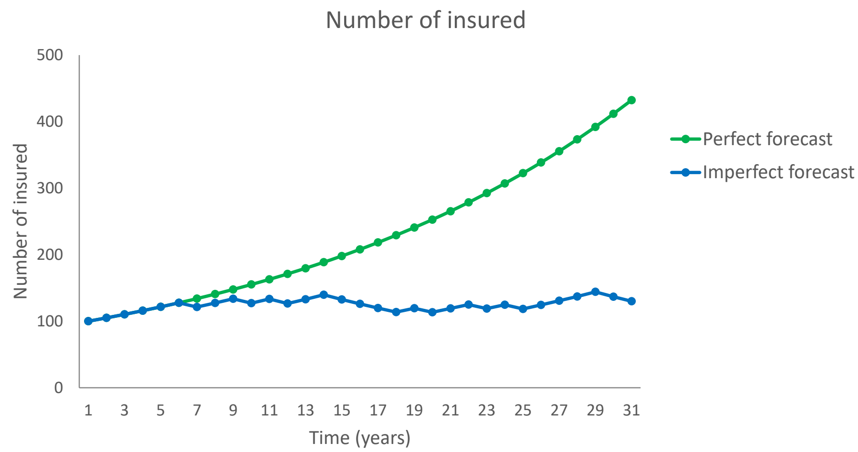

Fig. 6 The dynamics of the number of insured (green line: the hypothetical perfect forecast case; blue line: the case of forecast with low quality). See Sec. 6 for the details.

insurance. Strategies of individual producers might depend on availability of the weather index forecast and on its quality. The analysis performed suggests that the quality of the forecast would affect the optimal strategy to be selected by an insurance policy buyer under conditions of inevitable uncertainty. Modelling approaches developed in the present paper can support decisions relevant to design, successful implementation and subsequent scale-up of WII schemes in regions prone to agricultural droughts.

The analysis of adverse selection presented in Sections 4, 5 was made for a hypothetical scenario where only the policy buyers, but not the insurers, base their decision-making on available weather index forecast. An analogous scenario was considered in a study devoted to rainfall index insurance in the US, together with a more realistic scenario where both the insurers and the insured take into account weather index forecast [27]. As shown in [27], in the latter case adverse selection can be precluded by setting insurance premiums on the basis of forecast-conditional calculations. The generalization of the model developed in the present paper to account for the decision-making of insurers is left for further research.

Our findings are based on a simple conceptual model. In particular, a simple power law was chosen to parameterize the dependence of crop yield on the weather index. Also, the dynamics of both actual and forecasted weather index was, somewhat arbitrarily, represented by a very simple random process. Bringing more realism to the model, in particular, adopting the crop yield parametrisations from real-world field data, and deriving actual/forecasted weather indices from observations/forecasts of weather and climate variables, is planned for future research. Implementation of this program would make the proposed generic modelling scheme crop- and region- specific, which, as mentioned above, is a necessary prerequisite for successful real-world implementation of WII projects. Another interesting extension of this modelling scheme would be to consider the case where probabilistic forecasts are available to the insurers and the insured, providing not only the most likely values of the weather index, but additionally the confidence intervals.

In a simple WII model discussed in the present paper, the stochastic dynamics of actual weather index were simulated with a stationary random process. However, the climate is changing. In particular, climate model projections suggest that anthropogenic global warming will in future lead to wet areas getting wetter and to dry regions getting drier [2, 32, 33]. Generally, model projections reveal many drying areas in the low- and mid-latitudes, with a tendency for the subtropical (dry) zones to expand poleward [2, 33]. Consequently, future droughts will have more negative impacts on many economic activities, especially on agriculture, which is the most climate sensitive economic sector [34]. By generalizing the proposed modelling scheme to the case of non-stationary random processes, it would be possible to consider weather indices with statistically significant

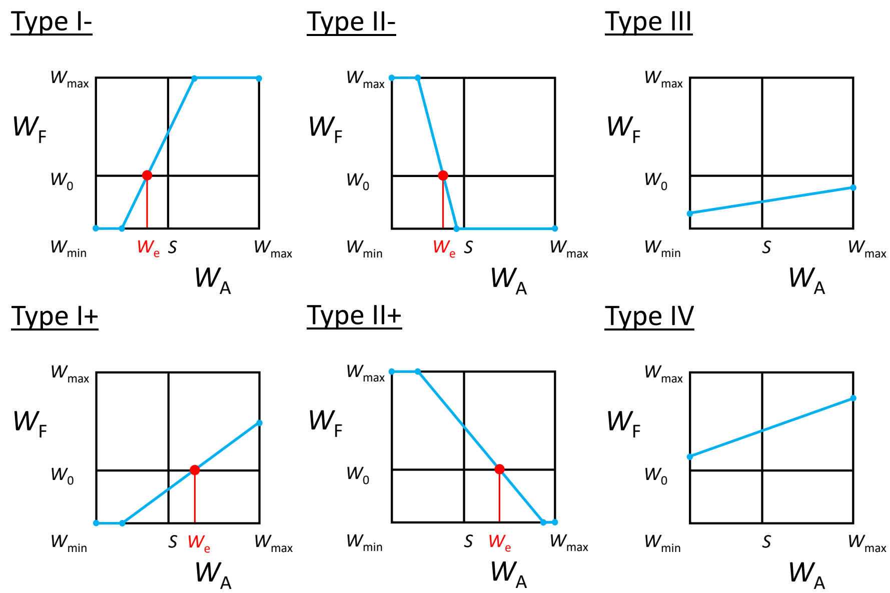

Fig. 7 A classification of six possible types of the model (29). See Appendix 7 for details.

trends that would be important for taking climate change effects into consideration. This would also likely shift the prospects for applications of this modelling scheme to the rapidly developing area of climate services. Nevertheless, the generalization of the proposed model to include climatic trends is also left for further research.

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Appendix. Calculations of surplus to producer's income

A. Case of model (29)

In the case of the model (29), the random error is absent, and the dependence of W_F on W_A is depicted by a straight line on the phase diagram (W_A, W_F). If the forecasted values go outside the bounds $W_{min} \le W_F \le W_{max}$, we assume that the forecast is adjusted to the closest bound (Figure 7).

The classification of all possible cases is shown in Figure 7 (Types I-, I+, II-, II+, III, and IV; see also Table 1). The values $W_A = S$ and $W_F = W_0$ break the phase plane (W_A, W_F) into four areas. For the purposes of

[b] The first results leading to this work have been presented in the conference paper: Kovalevsky, D.V., Máñez Costa, M., 2019: Modelling the Demand for Weather Index-Based Insurance Products in Regions Prone to Agricultural Droughts, the 59th ERSA Congress, 27-30 August 2019, Lyon, France, available at the congress website, URL:https://az659834.vo.msecnd.net/eventsairwesteuprod/production-ersa-public/fc71edc1c21c41e0ad1dba35d3e8019b.

Table 1 Classification of types of model (29).

Type	Interval where $F(w,W_0)$ varies when $W_F < 0$	Additional constraint
I-	$W_{\min} \le w \le W_e$	$W_e \le S$
I+	$W_{\min} \le w \le W_e$	$W_e > S$
II-	$W_e \le w \le W_{\max}$	$W_e \le S$
II+	$W_e \le w \le W_{\max}$	$W_e > S$
III	$W_{\min} \le w \le W_{\max}$	-
IV	$\emptyset$	-

classification and subsequent calculations, we introduce the entry/exit point W_e where the line (29) enters/exits the domain $W_F < 0$. Additionally, it is important for calculations whether $W_e < S$ or $W_e > S$.

We now apply Eq. (28) to calculate the surplus $\overline{\Delta I}$ for all possible types of the model (29).

Type I-.

In this case, the following expression for CDF holds:

$$F(w,W_0) = \begin{cases} (w - W_{\min})/\Delta W, & 0 \le w < W_e, \\ (W_e - W_{\min})/\Delta W, & W_e \le w \le W_{\max}. \end{cases} \tag{39}$$

In accordance with Eq. (28),

$$\overline{\Delta I} = \gamma[\int_{W_{\min}}^{W_e} \frac{w - W_{\min}}{\Delta W} dw + \int_{W_e}^{S} \frac{W_e - W_{\min}}{\Delta W} dw - (S - W_0)\frac{W_e - W_{\min}}{\Delta W}], \tag{40}$$

that yields

$$\overline{\Delta I} = \frac{\gamma}{2\Delta W}\left[2W_0 W_e - W_e^2 - W_{\min}\left(2W_0 - W_{\min}\right)\right]. \tag{41}$$

Properties of solution (41):

a) Eq. (41) is quadratic in W_e.

b) $\max \overline{\Delta I}(W_e)$ is reached at $W_e = W_0$, and is equal to

$$\overline{\Delta I}_{\max} = \frac{\gamma}{2\Delta W}\left(W_0 - W_{\min}\right)^2. \tag{42}$$

c) when $W_e > 2W_0 - W_{\min}$, $\overline{\Delta I} < 0$.

Type I+.

The CDF $F(w,W_0)$ is again given by Eq. (39), and

$$\overline{\Delta I} = \gamma[\int_{W_{\min}}^{S} \frac{w - W_{\min}}{\Delta W} dw - (S - W_0)\frac{W_e - W_{\min}}{\Delta W}]. \tag{43}$$

By making use of Eq. (13), we get

$$\overline{\Delta I} = \frac{\gamma}{\Delta W}\left(S - W_0\right)\left(W_{\max} - W_e\right). \tag{44}$$

Properties of solution (44):

a) Eq. (44) is linear in W_e.

b) $\max \overline{\Delta I}(W_e)$ is at $W_e = S$.

c) $\min \overline{\Delta I}(W_e)$ is at $W_e = W_{\max}$.

d) $\overline{\Delta I}(W_e)$ decreases with W_e.

e) $\overline{\Delta I} \ge 0$, and $\overline{\Delta I} = 0$ only when $W_e = W_{\max}$.

Type II-.

In this case,

$$F(w,W_0) = \begin{cases} 0, & 0 \le w < W_\mathrm{e}, \\ (w - W_\mathrm{e})/\Delta W, & W_\mathrm{e} \le w \le W_\mathrm{max}, \end{cases} \tag{45}$$

and

$$\overline{\Delta I} = \frac{\gamma}{2\Delta W}[W_\mathrm{e}^2 - 2W_0 W_\mathrm{e} + \{S^2 - 2(S - W_0)W_\mathrm{max}\}]. \tag{46}$$

Properties of solution (46):

a) Eq. (46) is quadratic in W_e.

b) $\min \overline{\Delta I}(W_\mathrm{e})$ is reached at $W_\mathrm{e} = W_0$.

c) $\overline{\Delta I}$ is decreasing with W_e at $W_\mathrm{min} \le W_\mathrm{e} \le W_0$, then $\overline{\Delta I}$ is increasing with W_e at $W_0 < W_\mathrm{e} \le S$.

Type II+.

$F(w,W_0)$ is again given by Eq. (45), and

$$\overline{\Delta I} = -\frac{\gamma}{\Delta W}(S - W_0)(W_\mathrm{max} - W_\mathrm{e}). \tag{47}$$

Properties of solution (46):

a) Eq. (47) is linear in W_e.

b) $\max \overline{\Delta I}(W_\mathrm{e})$ is reached at $W_\mathrm{e} = W_\mathrm{max}$.

c) $\overline{\Delta I}(W_\mathrm{e})$ is increasing with W_e.

d) always $\overline{\Delta I} \le 0$, and $\overline{\Delta I} = 0$ only when $W_\mathrm{e} = W_\mathrm{max}$.

Type III.

Now

$$F(w,W_0) = (w - W_\mathrm{min})/\Delta W, \quad W_\mathrm{min} \le w \le W_\mathrm{max}, \tag{48}$$

and

$$\overline{\Delta I} = 0. \tag{49}$$

Type IV.

Obviously,

$$F(w,W_0) = 0, \qquad W_\mathrm{min} \le w \le W_\mathrm{max}, \tag{50}$$

and

$$\overline{\Delta I} = 0. \tag{51}$$

B. Case of model (30)

The model (30) is a particular case of the model (29) fully analysed in Appendix A when $k = 1$, so we can now apply the results derived by us earlier.

Simple geometrical considerations suggest that in this case

$$W_\mathrm{e} = W_0 - \Delta, \tag{52}$$

and, dependent on Δ, the situation falls into one of the types defined in Appendix A (Figure 7; Table 1) in accordance with Table 2.

In case of Type I+, it follows from Eq. (44) that

$$\overline{\Delta I} = \frac{\gamma}{\Delta W}(S - W_0)(W_\mathrm{max} - W_0 + \Delta), \tag{53}$$

while in case of Type I- it follows from Eq. (41) that

$$\overline{\Delta I} = \frac{\gamma}{2\Delta W}[(W_0 - W_\mathrm{min})^2 - \Delta^2]. \tag{54}$$

Both $\overline{\Delta I}$ and its derivative are continuous at $\Delta = -(S - W_0)$, where Type I+ changes to Type I-.

Table 2 Possible types for model (30).

Interval where Δ varies	Type
$-\Delta W \leq \Delta \leq -(W_{\max} - W_0)$	III
$-(W_{\max} - W_0) < \Delta \leq -(S - W_0)$	I+
$-(S - W_0) < \Delta \leq W_0 - W_{\min}$	I-
$W_0 - W_{\min} < \Delta \leq \Delta W$	IV

C. Case of model (14)

To apply Eq. (28) to the model (14) under conditions described in Section 5, we notice from simple geometrical considerations that the CDF to be used in Eq. (28) is now equal to

$$F(w, W_0) = \frac{1}{2B\Delta W}\left[(d + k\overline{W})(w - W_{\min}) - \frac{k}{2}(w^2 - W_{\min}^2)\right], \tag{55}$$

and

$$F(W_0) = \frac{d}{2B}. \tag{56}$$

By performing integration in Eq. (28), we get the result (35).

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