Solutions:

1.1 Let $N = 12$. Table Exercise 1.1

q	$-\infty$	-1	0	1	$+\infty$
$m(q)$	1	2.968	3.521	4	7

Table Exercise **1.1**.

1.2

a) **D)**.
b) **A)**.
c) **B)**.

1.3

a) **B)**.
b) **C)**.
c) **A)**.
d) **C)**.
e) **A)**.

1.4

a) **B)**.
b) **B)**.
c) **A)**.

1.5

a) $\bar{x} = 187.5$
b) $m_e = 187.143$.
c) $s = 43.661$
d) $m_o = 184.211$.

1.6

a) **C)**.
b) **C)**.

1.7

a) **A)**.
b) **B)**.
c) **A)**.

1.8

a) **C)**.

b) **B)**.

1.9

a) $\bar{x} = 1442.46$.
b) $m_e = 2088.24$.
c) $s = 985.72$.
d) $m_o = 2056.34$.

1.10

a) **B)**.
b) **A)**.
c) **D)**.

1.11

a) $\bar{x} = 11.4$.
b) $s = 1.8$.
c) $D_8 = 13$.
d) $\gamma_2 = -0.742$. Platykurtic distribution.

1.12

a) **C)**.
b) **B)**.
c) **B)**.
d) **B)**.
e) **A)**.
f) **B)**.

1.13

a) $\bar{x} = 9.75$.
b) $m_e = 1.333$.
c) $m_o = 8.923$.
d) $s = 4.603$.

1.14

a) **C)**.
b) **B)**.

1.15

a) $\bar{x} = 9.106$.
b) $m_e = 8.868$.
c) $m_o = 8.820$.

1.16

a) $\bar{x} = 4.74$.
b) $m_g = 4.07$.
c) $m_h = 3.21$.
d) $s = 2.21$.

e) $F(x) = \begin{cases} 40, & 0 \leq x < 2 \\ 120, & 2 \leq x < 4 \\ 180, & 4 \leq x < 5 \\ 230, & 5 \leq x < 6 \\ 310, & 6 \leq x < 8 \\ 330, & 8 \leq x \leq 10 \end{cases}$

f) $m_o = 4.56$.
g) $m_e = 4.75$.

1.17

a) $\bar{x} = 8.3$.
b) $m_g = 6.0$.
c) $m_h = 3.4$.
d) $s = 5.4$.
e) $m_o = 2.1$.
f) $m_e = 7.1$.

1.18

a) $\bar{x} = 5.70$.
b) $m_g = 4.99$.
c) $m_h = 4.26$.
d) $s = 2.71$.
e) $m_o = 5.50$.
f) $m_e = 5.50$.

1.19

a) $\bar{x} = 3.70$.
b) $m_e = 4$.
c) $m_o = 9.016$.

1.20 $\rho_{XY} = -0.99872$.

1.21 $\rho_{XY} = -0.988$.

1.22
$\rho_{XY} = 0.9908$.

2.1 $P(A \cap \overline{B}) = P(A) - P(A \cap B) = P(A) - P(A)P(B) = P(A)\,[1 - P(B)] = P(A)P(\overline{B})$. Thus A and $\overline{B}$ are independent events.

2.2 $P(A) = P(B)$.

2.3
$P(A \cup B) = 1 - \alpha\beta$.

2.4
$P(A \cup B) = 0.5$.

2.5 $P(A \cup B) = 0.9$.

2.6

a) $P(A \cup B) = 0.75$.
b) $P(\overline{A} \cup \overline{B}) = 1$.
c) $P(\overline{A} \cap B) = 0.45$.

2.7

a) $P(B) = p = 0.3$.
b) $P(B) = p = 0.5$.

2.8

a) **A)**.
b) **B)**.
c) **A)**.

2.9

a) $P\left[(A \cap \overline{B})\left(\overline{A} \cap B\right)\right] = P(A) + P(B) - 2P(A \cap B)$.
b) No, since one must have $P(A \cap B) \leq P(B)$ and $P(A \cap B) \leq P(A)$ and this is false for the given values.

2.10

a) $P(A \cup B) = 0.775$.
b) $P(A \cup B) = 0.95$.

2.11

a) If A and B are mutually exclusive then $P(A \cup B) = P(A) + P(B)$, thus (i) $P(A \cap B) = 0$. For A and B to be independent, one must have $P(A \cap B) = P(A) \cdot P(B)$ and if $P(A) \neq 0$ and $P(B) \neq 0$, then (ii) $P(A \cap B) \neq 0$. Thus, we obtain (i) and (ii), which is a contradiction.
b) If A and B are independent then (i) $P(A \cap B) = P(A) \cdot P(B) \neq 0$, thus $P(A) \neq 0$ and $P(B) \neq 0$. For A and B to be mutually exclusive, one must have $P(A \cup B) = P(A) + P(B)$, thus (ii) $P(A \cap B) = 0$, which is a contradiction with (i).

2.12 $P(A \cup B) = \frac{7}{12}$.

2.13

a) **A)**.
b) **D)**.

2.14

a) No, $P(A \cap B \cap C) \neq P(A)P(B)P(C)$.
b) No, $A \cap B \neq \emptyset$.
c) $P(A|B) = 0.2$.

2.15 $P(A \cup B \cup C) = 3p_1 - 2p_2$.

2.16 $P(A \cup B \cup C) = \frac{5}{8}$.

2.17
$P(A \cup B \cup C) = 0.568$.

2.18 **i)**.

2.19

a) $p = 5/6$.
b) $p = 1/5$.

2.20

a) **C)**.
b) **A)**.

2.21

a) **B)**.
b) **A)**.

2.22

a) $P(A > B) = \frac{2}{3}$.
b) $P(B > C) = \frac{5}{9}$.
c) $P(C > A) = \frac{2}{3}$.
d) $P(A > B > C) = \frac{2}{9}$.

2.23 **C)**.

2.24

a) **A)**.
b) **B)**.
c) **A)**.

2.25

a) **D)**.
b) **B)**.

2.26

a) **B)**.
b) **D)**.
c) **A)**.

2.27

a) **A)**.
b) **A)**.
c) **D)**.
d) **C)**.

2.28 $P(C_1|\text{‘gold'}) = \frac{2}{3}$.

2.29

a) $p = 0.3456$.
b) $p = 0.3973$.

2.30

a) $P(A|L) = \frac{1}{1+p-p^2}$.
b) $P(L|A) = p$.
c) $P(L) = p^2(1+p-p^2)$.

2.31

a) $P(L) = 0.48$.
b) $P(A|L) = 0.4167$.

2.32

a) $P(E) = 0.163$.
b) $P(B|E) = 0.184$.

2.33

a) $P(D) = 0.0755$.
b) $P(B|D) = 0.166$.

2.34 $P(X = k) = \binom{20}{k} 0.092^k 0.982^{20-k},\ k = 0, 1, 2, \ldots, 20.$

2.35 0.311.

2.36

a) **B)**.
b) **C)**.

2.37

a) **B)**.
b) **C)**.
c) **D)**.
d) **A)**.
e) **D)**.

2.38

a) $p \simeq 0.1646$.
b) $q \simeq 0.1904$.
c) $r \simeq 0.1481$.

2.39

a) $p = 0.271$.
b) $p = 0.278$.

2.40

a) $p = 0.008$.
b) $q = 0.594$.
c) $r = 0.25$.

2.41

a) $P(X \geq 2) = 0.76672$,
b) $P(X \geq 1) = 0.77 \Leftrightarrow n > 2.877$.

2.42

a) $p_1 = 0.1^3$.
b) $p_2 = 2.97 \times 10^{-5}$.
c) $p_3 = 0.000996$.

2.43

a) $p \simeq 0.083$.
b) $q \simeq 0.3057$.
c) $r \simeq 0.1606$.

2.44 $E(X) = n \left[x \left(p + \frac{1-p}{k} \right) + y(1-p)\frac{k-1}{k} \right]$.

2.45

a) $P(D) = 0.074$.
b) $P(A|D) = 0.541$.

2.46

a) $\simeq 0.141$.
b) $\simeq 0.138$.

2.47

a) **D)**.
b) **B)**.

2.48

a) **C)**.
b) **A)**.

2.49

a) **A)**.
b) **C)**.

2.50

a) **C)**.
b) **A)**.
c) **B)**.

2.51

a) $p = \frac{n_A}{n_A+n_B+n_C}$.
b) $p = \frac{n_A}{n_A+n_B+n_C} + \frac{n_A-1}{n_A+n_B+n_C-1}$.
c) $p = \frac{n_A-1}{n_A+n_B+n_C-1}$.

2.52

a) 0.495.
b) 0.097.

2.53

a) $P(M) = 0.47$.
b) $P(C|M) = 0.319$.

2.54

a) **B)**.
b) **A)**.

2.55

a) **A)**.
b) **B)**

2.56 $n = 6, r = 3$.

2.57

x	1	2	3	4
$P(X=x)$	2/5	3/10	1/5	1/10

Table Exercise **2.59 a)**.

z	2	3	4
$P(Z=z)$	3/5	3/10	1/10

Table Exercise **2.59 c)**.

a) Table Exercise 2.59a).
b) $P(X \leq 2) = \frac{7}{10}$.
c) Table Exercise 2.59c).

d) $P(Z < 4) = \frac{9}{19}$.

2.58

a) $p = 0.887$.
b) $p = 0.3147$.

2.59

a) $p \simeq 0.931$.
b) $q = 1$.

2.60

a) **C)**.
b) **D)**.
c) **B)**.

2.61

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{8}x^2, & 0 \leq x < \frac{1}{2} \\ \frac{1}{8}x^2 + \frac{1}{8}, & \frac{1}{2} \leq x < 1 \\ \frac{1}{8}x^2 + \frac{3}{8}, & 1 \leq x < \frac{3}{2} \\ \frac{1}{8}x^2 + \frac{1}{2}, & \frac{3}{2} \leq x < 2 \\ 1, & x \geq 2 \end{cases}.$$

2.62 Let $P_n = \frac{1}{2} + (2p-1)^{n-1}\left(\theta - \frac{1}{2}\right)$. Then $\lim_{n\to\infty} P_n = \frac{1}{2}$, $|2p-1| < 1$.

2.63 $E\left[(X+1)^2\right] = \sigma^2 + (\mu+1)^2$.

2.64 Using the probability distribution, we obtain $P(|X-4.5| \geq 4) = 0.2$. From the Tchebycheff inequality, we obtain $P(|X-4.5| \geq 4) \leq \frac{1}{c^2}$. As $4 = c\sigma$, then $c \simeq 1.3928$, thus $\frac{1}{c^2} = 0.5155$. The Tchebycheff inequality provides a much higher value.

2.65 **B)**.

2.66 **B)**.

2.67

a) $E(Y) = \arctan(\mu) - \mu\left(\frac{\sigma}{1+\mu^2}\right)^2$.
b) $V(Y) = \left(\frac{\sigma}{1+\mu^2}\right)^2$.

2.68 $E(Y) = \mu^{n-2}\left(\mu^2 + \frac{n(n-1)}{2}\sigma^2\right)$.
$V(Y) = \left(n\mu^{n-1}\sigma\right)^2$.

2.69

a) $E(Y) = \tan(\mu)\left[1 + \left(\frac{\sigma}{\cos(\mu)}\right)^2\right]$.
b) $V(Y) = \left[\frac{\sigma}{\cos^2(\mu)}\right]^2$.

2.70 Let $\varepsilon > 0,\ f(x) > 0$ and $g(x)$ the probability density function of the random variable X.

$$\frac{1}{f(\varepsilon)}E\left[f(|X-\mu|)\right] = \frac{1}{f(\varepsilon)}\int_{-\infty}^{+\infty} f(|X-\mu|)g(x)dx =$$

$$= \int_{-\infty}^{\mu-\varepsilon} \frac{1}{f(\varepsilon)} f(|X-\mu|)g(x)dx + \int_{\mu-\varepsilon}^{\mu+\varepsilon} \frac{1}{f(\varepsilon)} f(|X-\mu|)g(x)dx +$$

$$+ \int_{\mu+\varepsilon}^{+\infty} \frac{1}{f(\varepsilon)} f(|X-\mu|)g(x)dx \geq$$

$$\geq \int_{-\infty}^{\mu-\varepsilon} g(x)dx + \int_{\mu+\varepsilon}^{+\infty} g(x)dx = P(X \geq \mu+\varepsilon) + P(X \leq \mu-\varepsilon)$$

Thus:

$$P(|X-\mu| \geq \varepsilon) \leq \frac{E\left[f(|X-\mu|)\right]}{f(\varepsilon)}.$$

2.71
$E(X) = \frac{1}{3}$. $V(X) = \frac{1}{9}$.

2.72

a) **A)**.
b) **A)**.
c) **C)**.
d) **A)**.
e) **A)**.
f) **D)**.
g) **B)**.

3.1 $P(A|B) = 0.659$, $P(B|A) = 1$.

3.2 $P(Y \leq 1) = 0.5$.

3.3

a) **A)**.
b) **B)**.

3.4 $P(X=k)=P(X=-k),\ k=1,2,\dots,n$. $P(|X|=k+1)=2/3P(|X|=k)$.
$P(X=0)=\frac{1}{5-4\left(\frac{2}{3}\right)^{n+1}}$.

3.5 $P\left(X=\frac{n}{n+1}\right)>0$ and $P\left(X=\frac{n+1}{n}\right)>0$.
$\sum\limits_{n=1}^{\infty}\left[P\left(X=\frac{n}{n+1}\right)+P\left(X=\frac{n+1}{n}\right)\right]=1$.

3.6

a) Table Exercise 3.6a).

k	1	2	3	4	5	6	7	8	9
$P(X\le x)$	0.301	0.477	0.602	0.699	0.778	0.845	0.903	0.954	1

Table Exercise **3.6a)**.

b) $E(X)\simeq 3.44$.
c) $V(X)\simeq 6.057$.

3.7

a) Table Exercise 3.7a).

Y_i	0	1	4	25	81
$P(Y_i)$	0.2	0.1	0.2	0.3	0.2

Table Exercise **3.7a)**.

$P(Y\ge 5)=0.5$.

b) Table Exercise 3.7b).

Z_i	-1	0	$\sqrt[3]{2}$	$\sqrt[3]{5}$	$\sqrt[3]{9}$
$P(Z_i)$	0.1	0.2	0.2	0.3	0.2

Table Exercise **3.7b)**.

$P(Z\le 0)=0.3$.

3.8

a) **B)**.
b) **A)**.
c) **C)**.

3.9

a) **B)**.
b) **C)**.
c) **D)**.

3.10

a) $P(1 < X \leq 3) = P(X = 2) + P(X = 3) = 1/6 + 1/3 = 1/2$.
b) $P(X = 0) = 1/3$, $P(X = 1) = 1/6$, $P(X = 2) = 1/6$, $P(X = 3) = 1/3$.

3.11 $P(X > 1) = 0.624$.

3.12

a) $P(X = 15) = 1.667 \times 10^{-7}$.
b) $P(X \geq 2) = 0.9308$.

3.13 Table Exercise 3.13.

k	0	1	2	3	4	5
$P(X = k)$	0.3277	0.4096	0.2048	0.0512	0.0064	0.0003

Table Exercise **3.13**.

3.14 Using the conditional probability, one obtains $P(X = k|\text{defective}) = \frac{k}{np}\binom{n}{k}p^k(1-p)^{n-k}$.
The non-conditional probability is $P(X = k) = \binom{n}{k}p^k(1-p)^{n-k}$.
If $\frac{k}{n} > p$, then the value of the conditional probability is higher.
If $\frac{k}{n} = p$, then the value of the conditional probability is equal to the value of the non-conditional probability.
If $\frac{k}{n} < p$, then the value of the conditional probability is smaller.

3.15 $n = 5$, $p = 0.2$.

3.16 $n = 3$, $p = \frac{2}{3}$.

3.17 $P(X > 3|X > 2) = 0.054$.

3.18

a) Table Exercise 3.18a).

k	0	1	2	3	4	5
p	0.0778	0.2592	0.3456	0.2304	0.0768	0.0102

Table Exercise **3.18a)**.

b) $F(x) = \begin{cases} 0, & x < 0 \\ 0.0778, & 0 \leq x < 1 \\ 0.3370, & 1 \leq x < 2 \\ 0.6826, & 2 \leq x < 3 \\ 0.9130, & 3 \leq x < 4 \\ 0.9898, & 4 \leq x < 5 \\ 1, & x \geq 5 \end{cases}$

c) $\mu = 2 = m_e = m_o$.

3.19

a) $P(X = 3|X > 1) = 0.1276$.
b) $n \leq 5$.

3.20

a) $X \sim Bi(10, 0.05)$. **A)**.
b) **C)**.

3.21 By the binomial distribution, we obtain $P(X = 10) = 2.457 \times 10^{-3}$. By the Poisson distribution, we obtain $P(X = 10) = 1.205 \times 10^{-12}$. The values od the probabilities are very different, since $n = 30$ is small. Better approximations are obtained for example for $n = 300$.

3.22

a) **A)**.
b) **B)**.

3.23 By the binomial distribution for $X \sim Bi(20, 0.05)$, one gets $P(X = 2) = 0.1887$. Using the Poisson distribution for $X \sim Po(1)$, one obtains $P(X = 2) = 0.1839$. The values differ in the third decimal place.

3.24 Using the binomial distribution, we obtain $P(X = 15) = 3.4529 \times 10^{-15}$. From the Poisson distribution, we get $P(X = 15) = 1.209 \times 10^{-14}$.

3.25

a) Using the probability function, $P(|X - E(X)| \geq 2) = 0.05792$. Using the Tchebycheff inequality $P(|X - E(X)| \geq 2) \leq \frac{1}{c^2}$. As $c\sigma = 2$, thus $\frac{1}{c^2} = 0.2$ and $P(|X - E(X)| \geq 2) \leq 0.2$. The value obtained by the Tchebycheff inequality is much higher that the one obtained using the probability function.

b) Using the expressions to approximate, we obtain $E(Y) \simeq 1.8$ and $V(Y) \simeq 3.2$. Without any ind of approximation, we obtain $E(Y) = 1.8$ and $V(Y) \simeq 6.432$. There is a significant error in the value of the variance.

3.26
Using the probability distribution, $E(Y) = 0.68$ and $V(Y) = 0.2176$. The approximate values are $E(Y) = 0.68$ and $V(Y) = 0.4608$. There is an error in the value of the variance.

3.27

a) $P(X = 0) = 0.018$.
b) $P(X' < 3) = 0.0138$.
c) $\alpha = 8.243$.

3.28 $k = 1,\ \alpha = 1$.

3.29

a) $P(Y=r)=\frac{e^{-\alpha}}{1-e^{-\alpha}}\frac{\alpha^r}{r!},\ r=1,2,\ldots$
b) $E(Y)=\frac{\alpha}{1-e^{-\alpha}}$.

3.30

a) $X \sim Po(2)$. $P(X \geq 1)=0.865$.
b) $X_1 \sim Po(4)$. $P(X_1 \leq 1)=0.0916$.

3.31

a) $p=0.2382$.
b) $q=0.7401$.

3.32 **A)**.

3.33 $n=5,\ N=10$.

3.34 $a=\frac{p}{1-p}$.

3.35 $p=\frac{1}{2}$.

3.36 $a=0 \Rightarrow b=\frac{1}{3}$, $a=1 \Rightarrow b=3$, $a=2,3,4 \Rightarrow b=0$.

3.37
$M_X(t)=\left(\frac{e^{t/2}-e^{-t/2}}{t}\right)^2$. $M_X'(t=0)=E(X)=0$.

3.38 X follows a binomial distribution with parameters $n=5$ and $p=0.20$.

3.39 $M_X(t)=\frac{1}{4}+\frac{1}{3}e^t+\frac{5}{12}e^{2t}$. $E(X^n)=\frac{1}{3}+\frac{5}{12}2^n,\ n=1,2,\ldots$

3.40 **A)**.

3.41 Table Exercise 3.41.

	$E(X\|y)$
$Y=1$	1.857
$Y=2$	1.833

Table Exercise **3.41**.

3.42

a) $E(X)=1.9$.
b) $E(Y|X \neq 2)=2.063$.
c) $P(Z=4)=0.35$.

d) Table Exercise 3.42d).

3.43

a) $P(X^2+Y^2=1)=0.40$.

b) Table Exercise 3.43b).

$W\backslash Z$	1	2	3	4	5	6
1	–	0.15	0.15	0.3	–	–
2	–	–	–	0.05	0.25	–
3	–	–	–	–	–	0.1

Table Exercise **3.42d)**.

$W\backslash Z$	0	1	2	3
0	0.10	0.40	0.15	–
1	0.15	–	0.10	–
2	–	0.05	–	0.05

.11Table Exercise **3.43b)**.

X	1	2	3
$P(X)$	0.35	0.25	0.40

.12Table Exercise **3.44**.

3.44

a) Table Exercise 3.44.

b) $P(Y=2|X=1)=0.314$.
c) $P(Z \geq 7|W=1)=P(X=3,Y=3)=0.25$.

3.45

a) Table Exercise 3.46 a).

$X\backslash Y$	$Y=-2$	$Y=0$	$Y=1$	$Y=3$	$P(X)$
$X=-1$	1/24	1/12	1/12	1/24	1/4
$X=0$	1/12	1/6	1/6	1/12	1/2
$X=1$	1/24	1/12	1/12	1/24	1/4
$P(Y)$	1/6	1/3	1/3	1/6	1

Table Exercise **3.46a)**.

b) X and Y are independent since $P(X=x_i,Y=y_j)=P(X=x_i)\times P(Y=y_j),\ \forall(x_i,y_j)$.

c) $P(Y>X)=\frac{1}{2}$.

3.46

a) Table Exercise 3.47a).

b) $P(X=0|Y>0)=\frac{5}{9}$.

	$Y=0$	$Y=1$	$Y=2$	$P(X)$
$X=0$	$1/6$	$1/3$	$1/12$	$7/12$
$X=1$	$1/12$	$1/6$	$1/6$	$5/12$
$P(Y)$	$1/4$	$1/2$	$1/4$	1

Table Exercise **3.47a)**.

c) $P(Y > x) = \begin{cases} 5/12, & x = 0 \\ 1/12, & x = 1 \end{cases}$

d) Table Exercise 3.47d).

Z	0	1
$P(Z = z_i)$	$2/3$	$1/3$

Table Exercise **3.47d)**.

3.47

a) Table Exercise 3.48a).

$X \backslash Y$	0	1	2	$P(X)$
0	$1/6$	$1/4$	$1/10$	$31/60$
1	$1/10$	$1/5$	$11/60$	$29/60$
$P(Y)$	$4/15$	$9/20$	$17/60$	1

Table Exercise **3.48a)**.

b) $P(X = 1|Y = 1) = 4/9$.

c) Table Exercise 3.48c).

$Z \backslash W$	0	1	2
0	$1/6$	$7/20$	$1/10$
1	0	$1/5$	$11/60$
2	0	0	0

Table Exercise **3.48c)**.

3.48

a) **A)**.

b) **D)**.
c) **B)**.
d) **B)**.

3.49

a) **B)**.
b) **C)**.
c) **D)**.
d) **A)**.
e) **B)**.
f) **B)**.

3.50

a) **A)**.
b) **B)**.
c) **B)**.
d) **D)**.

3.51

a) $E(X) = \frac{2}{3}$.
b) $V(X) = \frac{2}{9}$.
c) $\rho_{XY} = 0$.
d) $E(X|y=0) = \frac{2}{3}$, $E(X|y=1) = \frac{2}{3}$, $E(X|y=2) = \frac{2}{3}$.
e) $E\left[E(X|y_i)\right] = 1$.

3.52

a) $E(X) = 1.4$.
b) $V(X) = 1.66$.
c) $E(X|Y=3) = 2.357$.
d) $E(Y|X \geq 1) = 2.758$.
e) $E(X^2|Y=0) = 3.067$.
f) $E(X^2|Y=0) = -0.389$.

3.53

a) $E(X) = 0.13$.
b) $V(X) = 1.283$.
c) $\rho_{XY} = -0.048$.
d) $P(X=0|Y=1) = 0.3125$.
e) $E(X|Y=1) = -0.0625$.

f) Table Exercise 3.55f).

$W \setminus Z$	0	1	2
0	0.11	0.31	0.13
1	0.0	0.24	0.17
2	0.0	0.0	0.04

Table Exercise **3.55f)**.

3.54

a) $\rho_{XY} = 1$.
b) Table Exercise 3.56b).

$W \backslash Z$	0	1	2
-1	–	0.25	–
0	0.20	–	0.30
1	–	0.25	–

Table Exercise **3.56b)**.

3.55

a) Table Exercise 3.57a).

$Y \backslash X$	-1	0	1	2	$P(Y)$
-1	0.10	0.15	0.03	0.01	0.29
0	0.12	0.25	0.10	0.02	0.49
1	0.13	0.05	0.02	0.02	0.22
$P(X)$	0.35	0.45	0.15	0.05	1

Table Exercise **3.57a)**.

b) $P(X+Y^2 = 1) = 0.60$.
c) $E(X) = -0.10$, $E(Y) = -0.07$.
d) $V(X) = 0.69$, $V(Y) = 0.5051$.
e) $\rho_{XY} = -0.046$.
f) $E(X, y = 0) = 0.041$.
g) $P(X+Y = 0 | X-Y = 0) = 0.6757$.

h) Table Exercise 3.57h).

$W \backslash Z$	0	1	2	3
-1	0.10	–	–	–
0	0.25	0.27	–	–
1	0.02	0.15	0.16	–
2	–	0.02	0.02	0.01

Table Exercise **3.57h)**.

3.56

a) Table Exercise 3.58a).

$Y\backslash X$	-1	0	1	$P(Y)$
-1	0.10	0.10	0.20	0.40
1	0.25	0.05	0.30	0.60
$P(X)$	0.35	0.15	0.50	1.0

Table Exercise **3.58a)**.

b) $E(X) = 0.15$. $V(X) = 0.8275$.

c) $\rho_{XY} \simeq -0.09$.

d) Table Exercise 3.58d).

$W\backslash Z$	-1	0	1
-1	-		0.45
0	-	0, 10	0.05
1	0.10	-	0.30

Table Exercise **3.58d)**.

3.57

a) $E(X) = 2.05$.
b) $E(Y|X \neq 2) = 1.846$.
c) $V(X) = 0.6475$.
d) $\rho_{XY} \simeq -0.160$.
e) Table Exercise 3.59e).

$W\backslash Z$	2	3	4	5	6
$W = 0$	0.15	0	0.15	0	0.10
$W = 1$	0.	0.15	0	0.30	0
$W = 2$	0	0	0.15	0	0

Table Exercise **3.59e)**.

3.58

a) Table Exercise 3.60a).

b) $P(X^2 + Y^2 = 1) = 0.47$.
c) $\rho_{XY} = -0.104$.
d) $E(X, y = 1) = -0.55$.
e) $P(X + Y = 0|X - Y > -1) = 0.322$.

3.59

$Y\backslash X$	-1	0	1	$P(Y)$
-1	0.10	0.15	0.03	0.28
0	0.12	0.25	0.15	0.52
1	0.13	0.05	0.02	0.20
$P(X)$	0.35	0.45	0.20	1.0

Table Exercise **3.60a)**.

	X	0	1	2	3	$P(Y=y)$
	0	0.1	0.1	0.1	0.01	0.31
Y	1	0.01	0.01	0.2	0.01	0.23
	2	0.05	0.01	0.4	0	0.46
	$P(X=x)$	0.16	0.12	0.7	0.02	1

Table Exercise **3.61a)**.

a) Table Exercise 3.61a).

X	0	1	2	3
$P(X=x)$	0.16	0.12	0.7	0.02

Table Exercise **3.61a)**.

b) Table Exercise 3.61b).

Y	0	1	2
$P(Y=y)$	0.31	0.23	0.46

Table Exercise **3.61b)**.

c) Table Exercise 3.61c).

X	0	1	2	3
$P(X\|Y=2)$	0.109	0.022	0.87	0

Table Exercise **3.61c)**.

d) Table Exercise 3.61d).

Y	0	1	2
$P(Y \mid X \leq 1)$	0.714	0.071	0.214

Table Exercise **3.61d)**.

e) $P(X = 0 \mid Y < 2) = 0.204$.

f) Table Exercise 3.61f).

Z	0	1	2	3	4	5	6
$P(Z = z)$	0.37	0.01	0.21	0.01	0.4	0	0

Table Exercise **3.61f)**.

3.60

a) Table Exercise 3.62a).

X	1	2	3
$P(X)$	0.25	0.35	0.40

Table Exercise **3.62a)**.

b) Table Exercise 3.62b).

$Y \backslash X$	1	2	3
0	$2/5$	$4/7$	$3/8$
3	$3/5$	$3/7$	$5/8$

Table Exercise **3.62b)**.

c) $\rho_{XY} \simeq 0.0444$.

3.61

a) **A)**.
b) **A)**.

4.1

a) $M_X(t) = \frac{1}{4}\left(e^{-t/2} + e^{t/2}\right)$.

b) $M_X(t) = \frac{e^{3/2t}+e^{t/2}-e^{-t/2}-e^{-3/2t}}{4t}$, $t \neq 0$.

4.2
$M_X(t) = \frac{1}{4t}\left[\frac{5}{6}e^{4t} + \frac{1}{2}e^{2t} - \frac{1}{3}e^{t} - 1\right]$, $t \neq 0$.

4.3

a) $M_X(t) = \frac{1}{n}e^t\frac{1-e^{nt}}{1-e^t}$.
b) $M_X(t) = \frac{2}{t^2}\left[1+e^t(t-1)\right]$, $t \neq 0$.

4.4

a) $M_X(t) = \frac{1}{1-4t^2}$, $-\frac{1}{2} < t < \frac{1}{2}$.
b) $E(X) = M_X^{'}(0) = 0$. $V(X) = M_X^{''}(0) = 8$.

4.5

a) **A)**.
b) **B)**.
c) **B)**.

4.6

a) **A)**.
b) **B)**.

4.7

a) $M_X(t) = \frac{e^{at}}{1-(bt)^2}$, $|t| < \frac{1}{b}$.
b) $E(X) = M_X^{'}(0) = a$. $V(X) = M_X^{''}(0) - M_X^{'}(0) = 2b^2$.

4.8

a) $M_X(t) = e^{at}\left(\frac{e^{-b\frac{t}{2}}-e^{b\frac{t}{2}}}{bt}\right)^2$.
b) $E(X) = M_X^{'}(0) = a$.

4.9

a) **A)**.
b) **B)**.

4.10
The first derivative of $E|X-c| = \int\limits_{-\infty}^{\infty} |x-c|f(x)dx$ in order to c is equal to $2F(c)-1$, where $F(x)$ is the integral of $f(x)$. The minimum obeys $2F(c)-1=0$, so the miminum is at $c = m_e$.

4.11 $g(y) = e^{-y}$, $y > 0$.

4.12

a) **B)**.
b) **A)**.

4.13

a) $F(x) = \int\limits_0^x xe^{-x}dx = 1-(x+1)e^{-x}$.
b) $P(X > 2|X > 1) \simeq 0.552$.
c) $E(X) = 2$.

d) $V(X) = 2$.
e) $g(y) = \frac{1}{y^2}\ln(y),\ y > 1$.

4.14

a) $p = \frac{1}{a(\beta-\alpha)}(x_2 - x_1)$.
b) $q = \frac{1}{\beta-\alpha}\left(\sqrt{x_3} - \sqrt{x_4}\right)$.

4.15

a) $F(x) = 1 - e^{\mu x}(\mu x + 1),\ x \geq 0$.
b) $f(x) = \mu^2 x e^{-\mu x},\ x \geq 0$.
c) $P(X > 1/\mu) \simeq 0.736$.

4.16 $g(y) = \frac{2}{3},\ 0 \leq y \leq \frac{3}{2}$.

4.17

a) $f(x) = \frac{2}{a^2}x,\ 0 < x < a$.
b) $P(a/3 < X < a/2) = 0.139$.
c) $E(X) = \frac{29}{3}$.
d) $V(X) = \frac{a^2}{18}$.

4.18

a) $F(x) = \frac{x^2}{12} - \frac{1}{3}$.
b) $E(X) = \frac{28}{9}$.
c) $m_e = \sqrt{10}$.
d) $V(X) \simeq 0.32099$.

4.19

a) **B)**.
b) **A)**.
c) **B)**.

4.20

a) $P(X < 2) = \frac{7}{8}$.
b) $P(X > 1 | X < 2) = \frac{3}{7}$.
c) $E(X) = \frac{5}{4}$.
d) $V(X)$ does not exist.
e) $g(y) = \begin{cases} \frac{1}{3}y^{-\frac{5}{3}}, & y \geq 1 \\ \frac{1}{6}y^{-\frac{2}{3}}, & 0 \leq y \leq 1 \end{cases}$.

4.21 $P(A|B) = \frac{1}{2}$. $P(B|A) = \frac{1}{2}$.

4.22 $a = 5/64$.

4.23 $g(y) = \begin{cases} \frac{3}{8\sqrt{y}}, & 0 < y \leq 1 \\ \frac{1}{8\sqrt{y}}, & 1 \leq y < 4 \\ 0, & y \leq 0 \vee y > 4 \end{cases}$.

4.24

a) $g(y) = \begin{cases} \frac{2}{\pi}\frac{1}{1+y^2}, & 0 < y \le c \\ 0, & y > c \\ \left(1 - \frac{2}{\pi}\arctan(c)\right) f(y), & y = 0 \end{cases}.$

b) $E(Y^2) = \frac{c}{\arctan(c)} - 1.$

4.25

a) $k = e.$
b) $E(X) = e - 1.$
c) $V(X) = \frac{(e-1)(3-e)}{2}.$
d) $P(X > (k+1)/2 | X < (k+2)/3) = 0.$

4.26

a) $k = 2.$
b) $E(X) = 2\ln(2).$
c) $V(X) \simeq 0.0782.$
d) $P(X > (k+2)/3 | X < (k+4)/3) = \frac{1}{2}$. It equals the non conditional probability $P((k+2)/3 < X < (k+4)/3)$.

4.27

a) $P(X > 2 | X < 3) = \frac{\left(\frac{2}{3}\right)^{1-n} - 1}{\left(\frac{1}{3}\right)^{1-n} - 1}.$
b) $E(X) = \frac{n-1}{n-2}$, $n > 3$. $V(X) = \frac{n-1}{(n-3)(n-2)^2}.$
c) $E(Y) \simeq \frac{1}{2-n} + \frac{\alpha(\alpha-1)}{2}\frac{n-1}{n-3}\frac{1}{(2-n)^{\alpha}}$. $V(Y) \simeq \frac{\alpha^2}{(2-n)^{2\alpha}}\frac{n-1}{n-3}.$

4.28

a) $f(x) = \frac{1}{(1+x)^2}$, $x \ge 0.$
b) $a = 1.$
c) $P(X \le 1/2 | X \le 1/3) = 1$. $P(X \le 1/3 | X \le 1/2) = \frac{3}{4}.$
d) $E(X)$ does not exist.

4.29 $g(y) = \begin{cases} \frac{1}{2\sqrt{y}}, & 0 < \sqrt{y} < \frac{1}{2} \\ \frac{1}{4\sqrt{y}}, & \frac{1}{4} < \sqrt{y} < \frac{9}{4} \end{cases}.$

4.30

a) $E(X) = -\frac{ab^4}{12}.$
b) $V(X) = -\frac{ab^5}{720}(5ab^3 + 36).$

4.31 $g(y) = \begin{cases} \frac{3}{4}, & |y| \le 1 \\ \frac{1}{4}, & 1 < |y| \le 2 \end{cases}.$

4.32

a) $E(X) = \sqrt{\frac{\pi}{2}}a.$
b) $V(X) = \left(2 - \frac{\pi}{2}\right)a^2.$
c) $P(X > a) = \frac{1}{\sqrt{e}}.$

4.33

a) $E(X) = \frac{1}{\alpha^{\frac{1}{\beta}}}\Gamma\left(\frac{1}{\beta} + 1\right).$
b) $V(X) = \frac{1}{\alpha^{\frac{2}{\beta}}}\left\{\Gamma\left(\frac{2}{\beta} + 1\right) - \left[\Gamma\left(\frac{1}{\beta} + 1\right)\right]^2\right\}.$

4.34 $E(X)$ does not exist, since the integral diverges.

4.35

a) $f(x) = F^{'}(x) f(x) = \frac{\alpha\beta}{\alpha-\beta}\left(x^{\beta-1} - x^{\alpha-1}\right) =.$
b) $E(X) = \frac{\alpha\beta}{(\alpha+1)(\beta+1)}$.
c) $V(X) = \alpha\beta \frac{(\alpha+1)^2+(\beta+1)^2-1}{(\alpha+1)^2(\alpha+2)(\beta+1)^2(\beta+2)}$.
d) $m_e = \left[1 - \sqrt{\frac{1}{2}}\right]^{\frac{1}{\beta}}$.

4.36

a) **A)**.
b) **D)**.
c) **D)**.

4.37

a) **D)**.
b) **A)**.
c) **C)**.
d) **A)**.

4.38

a) $\alpha = 1 + \frac{1}{\beta}$.
b) $g(y) = \begin{cases} \frac{1}{\alpha\gamma}, & 0 \leq y \leq \gamma x \\ 0, & y > \gamma\alpha \vee y < 0 \end{cases}$.

4.39 **B)**.

4.40 $g(y) = \begin{cases} \frac{1}{2\alpha}, & -\beta < y < \beta \\ \frac{\alpha-\beta}{2\alpha} f(y-\beta), & y > \beta \\ \frac{\alpha-\beta}{2\alpha} f(y+\beta), & y < -\beta \end{cases}$

4.41

a) $\alpha = 0.536$, $\beta = 7.464$.
b) Impossible, since $\alpha < 0$ and $\beta > 8$.

4.42 $m(q) = \left[\frac{1^q+2^q+\ldots+n^q}{n}\right]^{\frac{1}{q}}$. For $q = 1$, we obtain $m(1) = \frac{n+1}{2} = E(X)$. For $q = -1$, we obtain $m(-1) = \frac{n}{1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}}$.

4.43 $Y = \ln(X^2) \Rightarrow e^{Y/2} = x \Rightarrow \frac{dx}{dt} = \frac{1}{2}e^{Y/2}$. Then, the probability density function is

$$g(y) = 1 \cdot \left|\frac{1}{2}e^{Y/2}\right| = \frac{1}{2}e^{Y/2},\ y \leq 0$$

4.44

a) $E(Y) = 11$.
b) $V(Y) = 3$.

4.45 $s(z) = \begin{cases} 0, & z+0.01 < 0 \\ 50\left(1 - e^{-0.2(z+0.01)}\right), & -0.01 < z < 0.01 \\ 50\left(e^{-0.2(z-0.01)} - e^{-0.2(z+0.01)}\right), & z > 0.01 \end{cases}.$

4.46

a) **A)**.
b) **B)**.

4.47 $P(1/a < X < 2/a) \simeq 0.233$.

4.48

a) **A)**.
b) **D)**.

4.49

a) $E[(X-a)^2] = \frac{1}{\alpha^2} + \left(\frac{1}{\alpha} - a\right)^2$.
b) $E(|X-b|) = \frac{2e^{-\alpha b}-1}{\alpha} + b$.

4.50 $P(1.2 < X \leq 4.3) = 0.5581$.

4.51 $P(4 \leq X \leq 8) = 0.2286$.

4.52 $a = \mu + 0.46\sigma$.

4.53 $P(4 \leq X < 7) = 0.2039$.

4.54 $M_x(t) = e^{\left(\mu t + \frac{1}{2}\sigma^2 t^2\right)}$.
$M_x'''(t) = \mu\left(\mu^2 + 3\sigma^2\right)$.

4.55 **A)**.

4.56 $P(X - X^2 > 0) = P(0 < X < 1) = 0.3413$.

4.57

a) Let $X \sim N(1.70, 0.0025)$. $P(X = 1.80) = 0$.
b) $P(X > 1.80|X > 1.75) = 0.1437$.
c) $P(1.60 \leq X \leq 1.80) = 0.9544$. So, 95.44%.

4.58

a) $X \sim N(10, 0.04)$. $P(X > 10.5) = 0.0062$.
b) $E(L) = 2.938$ *u.m.*.
c) $\sigma = 0.4$ *u.m.*

4.59

a) **A)**.
b) **C)**.
c) **A)**.

4.60 **C)**.

4.61 $\mu = \frac{b+a}{2} - \frac{\sigma^2}{b-a} \ln\left(\frac{C_1+C_3}{C_1+C_2}\right)$.

4.62 Using the binomial distribution, we obtain $P(X=3)=0.1823$. From the Poisson distribution, one gets $P(X=3)=0.1804$. Using the gaussian distribution, we obtain $P(X=3)=0.2171$.

4.63
Using the binomial distribution, we obtain $P(X=2)=0.0836$. From the Poisson distribution, one gets $P(X=2)=0.0842$. Using the gaussian distribution, we obtain $P(X=2)=0.0732$.

4.64 Using the binomial distribution, we obtain $P(X=4)=0.13383$. From the Poisson distribution, one gets $P(X=4)=0.13385$. Using the gaussian distribution, we obtain $P(X=4)=0.1184$.

4.65

a) $P(-\sigma+\mu \le X \le \sigma+\mu \mid -2\sigma+\mu \le X \le 2\sigma+\mu)=0.715$.
b) $P(-\sigma+\mu \le X \le \sigma+\mu \mid -2\sigma+\mu \le X \le 2\sigma+\mu)=0.910$.

4.66

a) $a=5$, $b=3$.
b) $a \simeq 5.672$, $b \simeq 3.957$.

4.67 $p=\frac{1}{n}$.

4.68 $x_\alpha = \frac{1}{\lambda}\ln\left(\frac{1+\alpha}{\alpha}\right)$.

4.69 $t=1.7709$.

4.70 **B)**.

4.71 $x=1.7531$.

4.72

a) $t_{20,0.9}=1.3253$.
b) $t_{20,0.25}=-0.6870$.
c) $P(X<1)\simeq 0.824$.

4.73 Using the t-student distribution with $n=1$ degrees of freedom, we obtain

$$g(y)=\frac{1}{\Gamma(1/2)\Gamma(1/2)}y^{-1/2}(1+y)^{-1}$$

Using the F distribution with $n_1=1$ and $n_2=1$ degrees of freedom, we obtain

$$f(x)=\frac{1}{\Gamma(1/2)\Gamma(1/2)}x^{-1/2}(1+x)^{-1}$$

The two expressions are identical.

4.74

a) $x=2.6$.
b) $x=14.845$.

4.75

a) $\alpha \simeq 7.05$.
b) $\alpha \simeq 5.348$.

4.76

a) $a = 7.612$, $b = 14.141$.
b) $a = -0.444$, $b = 1.016$.

4.77

a) $a = 24.996$.
b) $a = 1.7531$.
c) $a = 3.33$.

4.78

a) $a = 37.7$.
b) $a = 1.708$.
c) $a = 2.71$.

4.79

a) **A)**.
b) **C)**.
c) **D)**.
d) **A)**.

4.80 $M_X(t) = (1-2t)^{-\frac{n}{2}}$.

4.81

a) $\chi^2_\alpha = 4.865$. $P(X \leq 20.5) =$
b) For $n = 15$, $P(|Z| < 5) \simeq 0.350$. For $n = 2$, $P(|Z| < 5) \simeq 0.998$.

4.82 $a = 2.833$, $b = 4.255$.

4.83 $M_X(t) = (1-2t)^{-\frac{n}{2}}$. $E(X) = M_X'(0) = n$. $V(X) = M_X''(0) - (E(X))^2 = 2n$.

4.84 $F(x) = 1 - e^{-2x}(1 + 2x + 2x^2)$.

4.85 **B)**.

4.86 $P(1 \leq X \leq 4) = 0.1411$.

4.87 $F(X) = P(X \leq x) = 1 - \sum_{k=0}^{2} e^{-4x} \frac{(4x)^k}{k!}$, $x > 0$.

4.88 $M_Z(t) = \left(\frac{0.2}{0.2-t}\right)^{20}$. Z follows a gamma distribution with parameters $\alpha = 0.2$ and $r = 20$. $E(Z) = \frac{r}{\alpha} = 100$, $V(Z) = \frac{r}{\alpha^2} = 500$.

4.89

a) **A)**.
b) **D)**.
c) **C)**.

4.90

a) $P(X > 2) \simeq 0.6767$.
b) $E(X) = 3$.
c) $V(X) = 3$.

4.91

a) **B)**.

b) **D)**.

4.92

a) $M_X(t) = (e^{+}1)^{\alpha}$.
b) $E(X) = 2^{\alpha-1}\alpha$. $E(X)$ increases with α.
c) $V(X) = \alpha(1+\alpha-2^{\alpha}\alpha)2^{\alpha-2}$. $V(X)$ is maximum for $\alpha = 0.6686$.

4.93

a) $P(Z>0) = \frac{1}{2}$.
b) $P(Z=0) = 0$, since Z is a continuous random variable.
c) $h(z) = \frac{1}{\pi}\frac{1}{z^2+1}, \ -\infty < z < +\infty$.

4.94 X and Y are independent random variables. Let $h(y) = \frac{8}{3}$ and $g(x|y) = \frac{3}{8}x^2$. Thus $E(X|Y) = \frac{3}{2}$.

4.95

a) $F(x) = \frac{1}{\pi}\arctan(x) + \frac{1}{2}$.
b) $h(z) = \frac{2}{\pi}\frac{1}{z^2+4}$.

4.96 $k(u,v) = ue^{-u}, \ u \geq 0, 0 \leq v \leq 1$.
$s(u) = ue^{-u}, \ u \geq 0$, $r(v) = 1, \ 0 \leq v \leq 1$, thus $k(u,v) = s(u)\cdot r(v)$, that proves that U and V are independent.

4.97

a) $k(u,v) = \frac{2}{u^3 v}, v \leq u^2$.
b) $s(u) = \frac{2}{u^3}\ln(u^2), \ u \geq 1$. $r(v) = \frac{1}{v^2}, \ v \geq 1$.

4.98 $h(u) = \frac{u}{\sigma^2}e^{-\frac{u^2}{2\sigma^2}}, \ 0 \leq u < +\infty$.

4.99

a) $F(x,y) = x^2y^2, \ 0<x<1, 0<y<1$. $F(x,y) = y^2, \ x>1, 0<y<1$. $F(x,y) = x^2$, $0<x<1, y>1$.
b) 0.3078.
c) $g(x) = 2x, \ 0<x<1$. $h(y) = 2y, \ 0<y<1$.
d) $P(X+Y>1) = \frac{5}{6}$.
e) $\rho_{XY} = 0$, since X and Y are independent random variables.
f) $\psi(z,w) = 2(w-\sqrt{z})$.

4.100

a) **C)**.
b) **C)**.

4.101 Let $V(XY) = E[(XY)^2] - [E(XY)]^2$. If X and Y are independent, then $E(XY) = E(X)E(Y)$. Thus, $E[(XY)^2] = E(X^2)E(Y^2)$. We obtain:

$$\begin{aligned} V(XY) &= E(X^2)E(Y^2) - [E(X)E(Y)]^2 = \ldots = \\ &= V(X)V(Y) + V(X)[E(Y)]^2 + V(Y)[E(X)]^2 \geq V(X)V(Y) \end{aligned}$$

4.102 **C)**.

4.103

a) $c \simeq 1.7185$.
b) $f(x) = c\left(1 - \frac{27}{x}\right)$
c) $E(X) = 18c$.
d) $E(XY) = 522c$.

4.104

a) $g(x) = \frac{x}{2},\ 0 \le x \le 2$ and $h(y) = \frac{2y}{15},\ 1 \le y \le 4$, thus $f(x,y) = g(x) \cdot h(y)$.

b) $F(x,y) = \begin{cases} \frac{x^2}{4}, & 0 \le x \le 2, y \ge 4 \\ \frac{(y^2-1)}{15}, & x \ge 2, 1 \le y < 4 \\ \frac{x^2(y^2-1)}{60}, & 0 \le x < 2, 1 \le y < 4 \\ 1, & x \ge 2, y \ge 4 \\ 0, & \text{o.v.} \end{cases}$

4.105

a) $p = 0.75$.
b) $q \simeq 0.8735$.

4.106

a) $h(y) = \frac{1}{2}\left(1 + \frac{1}{y \ln 2}\right),\ \ 1 \le y \le 2$.
b) $P(X \ge Y) = 0$.
c) $g(x|y) = \frac{2x + \frac{1}{\ln 2}}{1 + \frac{1}{y \ln 2}}$. Thus $P\left(X > \frac{1}{2} | Y = \frac{3}{2}\right) \simeq 0.627$.

4.107

a) $k = 65.0$.
b) $h(y) = k\left[\cos y - \cos y + 1/4\right],\ 0 < y < 1/4$.
c) $0.00646k \simeq 0.420$.

4.108

a) $g(x_1, x_2) = x_1 + x_2,\ 0 < x_1 < 1,\ 0 < x_2 < 1$.
b) $h(x_2) = 1/2 + x_2,\ 0 < x_2 < 1$.
c) $F(X_2) = P(X_2 \le x_2) = \begin{cases} 0, & x_2 < 0 \\ 1/2 x_2(1 + x_2), & 0 < x_2 < 1 \\ 1, & x_2 > 1 \end{cases}$.

4.109

a) $g(x) = \frac{4}{3}x^3 - x^2 + 1,\ 0 \le x \le 1$. $h(y) = \frac{y}{3} + \frac{y^2}{4} - \frac{y^3}{12},\ 0 \le y \le 2$.
b) $P(A) \simeq 0.1412$.

4.110

a) **A)**.
b) **B)**.
c) **D)**.

4.111

a) **B)**.
b) **A)**.
c) **D)**.
d) **A)**.

4.112

a) $g(x) = \frac{3}{2}\sqrt{x},\ 0 < x < 1.$
b) $h(y) = 3(1-\sqrt{y}),\ 0 < y < 1.$
c) $g(y|x = 1/2) = 2,\ 0 < y \le \frac{1}{2}.$
d) $h(x|y = 1/2) = \frac{1}{(2-\sqrt{2})\sqrt{x}},\ \frac{1}{2} < x < 1.$
e) $E(X) = \frac{3}{5}.$
f) $V(X) = \frac{12}{175}.$
g) $\rho_{XY} \simeq 0.569.$

4.113

a) $g(x) = 2x,\ 0 < x < 1.$ $h(y) = 2(1-y),\ 0 < y < 1.$
b) $E(X) = \frac{2}{3}.$ $E(Y) = \frac{1}{3}.$
c) $V(X) = \frac{1}{18}.$ $V(Y) = \frac{1}{18}.$
d) $\rho_{XY} = \frac{1}{2}.$
e) $E(X|Y) = \frac{1+y}{2},\ 0 < y < 1.$ $E(Y|X) = \frac{1}{2}x.$

4.114

a) $c = 3.$
b) $F(x,y) = \begin{cases} 0, & x < 0,\ y < 0 \\ 3xy, & 0 \le x \le 1,\ 0 \le y \le 1 \\ 3x, & 0 \le x \le 1,\ y > 1 \\ 3y, & x > 1,\ 0 \le y \le 1 \\ 1, & x > 1,\ y > 1 \end{cases}.$
c) $g(x) = 3\left(\sqrt{x} - x^2\right),\ 0 \le x \le 1.$ $h(y) = 3\left(\sqrt{y} - y^2\right),\ 0 \le y \le 1.$
d) $E(X) = 0.45,\ E(Y) = 0.45.$
e) $V(X) = 0.0546,\ V(Y) = 0.0546.$
f) $\rho_{XY} = 0.87.$
g) $E(X|Y) = \frac{\sqrt{y}}{2}\frac{1-y^3}{1-y^{3/2}},\ 0 \le y \le 1.$

4.115

a) $C = 4.$
b) $g(x) = \begin{cases} 4x, & 0 < x \le \frac{1}{2} \\ 4(1-x), & \frac{1}{2} < x < 1 \end{cases}.$ $h(y) = 2y,\ 0 < y < 1.$
c) $g_{X|y}(x) = \frac{2}{y},\ \frac{y}{2} < x < y,\ 0 < y < 1.$
d) $E(X|Y) = \frac{3}{4}y,\ 0 < y < 1.$
e) $\rho_{XY} = \frac{\sqrt{3}}{2}.$

5.1 $E(\bar{X}) = 1.0,\ V(\bar{X}) = 0.08.$

5.2 $P(S = 3) = 0.00715.$

5.3

a) $P(Y = 5) = 0.1008.$
b) $P(Y = 5) = 0.1173.$

5.4

i	$F_i = x_i$	$y_i = 3F_i - 1$
1	0.351	0.053
2	0.645	0.935
3	0.968	1.904

Table Exercise **5.4a)**.

a) $g(y) = \frac{1}{3}$, $-1 < y < 2$. Table Exercise 5.4a).

$$F(y) = \begin{cases} 0, & y < -1 \\ \frac{1}{3}(y+1), & -1 \le y \le 2 \\ 1, & y > 2 \end{cases}.$$

b) $P(0) = \binom{2}{0} 0.2^0 0.8^2 \simeq 0.64$, $P(1) = 0.2^1 0.8^1 \simeq 0.32$, $P(2) = 0.2^2 0.8^0 \simeq 0.04$.

$$F(y) = \begin{cases} 0, & k < 0 \\ 0.64, & 0 \le k < 1 \\ 0.96, & 1 \le k < 2 \\ 1, & k \ge 2 \end{cases}.$$

Table Exercise 5.4b).

i	$F_i = x_i$	$y_i = 3F_i - 1$
1	0.351	0
2	0.645	1
3	0.968	2

Table Exercise **5.4b)**.

5.5 $\psi(y_2) = -\ln(y_2)$, $0 < y_2 < 1$.

5.6 $V(\bar{X}) = \frac{1}{12n}$.

5.7 $\sigma = \sqrt{\frac{1}{27n}}$.

5.8

a) $x_i = 2 + 3\phi^{-1}(y_i)$. Table Exercise 5.8a).

y_i	$\phi^{-1}(y_i)$	x_i
0.363	-0.35	0.95
0.648	0.38	3.14
0.881	1.18	5.54
0.719	0.58	3.74

Table Exercise **5.8a)**.

b) $x = \frac{1}{\alpha} \ln\left(\frac{1}{1-y}\right)$. Table Exercise 5.8b).

0.363	0.15
0.648	0.35
0.881	0.71
0.719	0.42

Table Exercise **5.8b)**.

5.9

a) Table Exercise 5.9a).

i	x_i	y_i
1	0.024	1
2	0.152	1
3	0.565	3
4	0.738	4

Table Exercise **5.9a)**.

b) Table Exercise 5.9b).

i	x_i	y_i
1	0.024	0.020
2	0.152	0.137
3	0.565	0.694
4	0.738	1.116

Table Exercise **5.9b)**.

5.10 $M_X'(0) = 0, M_X''(0) = 1 = \frac{2!}{2^1 1!}, M_X'''(0) = 0, M_X^{iv}(0) = 3 = \frac{4!}{2^2 2!}, M_X^{v}(0) = 0, M_X^{vi}(0) = \frac{6!}{2^3 3!}$.

5.11 $g(s) = \sqrt{\frac{2}{\pi}} \frac{s^2}{\sigma^3} e^{-\frac{s^2}{2\sigma^2}}$, $s \geq 0$.

5.12 $j(s) = \sqrt{\frac{2}{\pi}} \frac{s^2}{\sigma^3} e^{\left(-\frac{1}{2}\frac{s^2}{\sigma^2}\right)}$. Maxwell distribution.

5.13 $P(Y > 3.94) = 0.95$.

5.14

a) **B)**.
b) **C)**.

5.15 $P(Y > 3.94) = 0.95$.

5.16

a) **A)**.
b) **B)**.

5.17

a) **C)**.
b) **A)**.
c) **C)**.

5.18 $a = 14.845$.

5.19 $a = 0.01$.

5.20 **A)**.

5.21
$a \simeq 7.779$.

5.22

a) **C)**.
b) **A)**.
c) **B)**.
d) **D)**.
e) **A)**.
f) **A)**.

5.23

a) **D)**.
b) **A)**.
c) **B)**.

5.24

a) $P(X = k) = \left\{1 - \left[1 - (1-p)^k\right]^n\right\} - \left\{1 - \left[1 - (1-p)^{k-1}\right]\right\}^n$.
b) $P(X = m) = [1 - (1-p)^m]^n - \left[1 - (1-p)^{m-1}\right]^n$.

5.25 $P(S \leq 0.50) = 0.00282$. $a = 4.8619$.

5.26 $P(S \leq 0.50) = 0.000297$. $a \simeq 10.6962$.

5.27 $C = \frac{n_2 - 1}{n_1 - 1\left(\frac{\sigma_Y}{\sigma_X}\right)^2}$. It is an F distribution with $(n_1 - 1)$ and $(n_2 - 1)$ degrees of freedom.

5.28 Table Exercise 5.28.
By the deMoivre-Laplace Theorem, we obtain $P(\overline{X} = 0.7) = 0.2085$.
Using the Central Limit Theorem (CLT), we get $P(\overline{X} = 0.7) = 0.1539$. As n is small, the approximation using the CLT is not good. The deMoivre-Laplace Theorem is more accurate.

5.29 $n_1 = 15 \Rightarrow P(18 < \overline{X} < 25) = 0.4857$. $n_2 = 150 \Rightarrow P(18 < \overline{X} < 25) = 0.8907$. The higher the value of n the better is the approximation of $\overline{X}$ to a gaussian distribution $N(20, \frac{20^2}{n})$.

5.30 Table Exercise 5.30.
By the Central Limit Theorem, as n grows, the distribution of Y_n approaches the normal distribution.

k	$\bar{X}$	$P(\bar{X})$	approx value
0	0	0.4^{10}	0.00010
1	0.1	$\binom{10}{1}0.4^9 0.6$	0.00157
2	0.2	$\binom{10}{2}0.4^8 0.6$	0.01062
3	0.3	$\binom{10}{3}0.4^7 0.6$	0.04247
4	0.4	$\binom{10}{4}0.4^6 0.6$	0.11148
5	0.5	$\binom{10}{5}0.4^5 0.6$	0.20066
6	0.6	$\binom{10}{6}0.4^4 0.6$	0.25082
7	0.7	$\binom{10}{7}0.4^3 0.6$	0.21499
8	0.8	$\binom{10}{8}0.4^2 0.6$	0.12093
9	0.9	$\binom{10}{9}0.4 0.6$	0.04031
10	1.0	0.6^{10}	0.00605

Table Exercise **5.28**.

Y_1	0	1
$P(Y_1)$	0.60	0.40

Table Exercise **5.30**.

Y_2	0	1/2	1
$P(Y_2)$	0.36	0.48	0.16

Table Exercise **5.30**.

Y_3	0	1/3	2/3	1
$P(Y_3)$	0.216	0.432	0.288	0.064

Table Exercise **5.30**.

Y_3	0	1	2	3
$P(Y_3)$	0.1664	0.4084	0.3341	0.0911

Table Exercise **5.31a)**.

5.31

a) $P(X=0)=0.55$, $P(X=1)=0.45$. $Y \sim Bi(3,0.45)$. Table Exercise 5.31a).

b) By the Central Limit Theorem, one gets $P(Y_{10}=6)=P(5.5<Y_{10}<6.5)=0.1591$. Using the binomial distribution, we obtain $P(Y_{10}=6)=\binom{10}{6}0.45^6 0.55^4=0.1596$. There is an error in the fourth decimal place.

5.32

a) $P(|X_A|>1)=0.6170$. $P(|X_B|>1)=0.5228$.

b) $X_A-X_B \sim N(-1,5)$. $P(|X_A-X_B|>1)=0.6867$.

c) By the Central Limit Theorem, the sum of 300 independently measurements S_{300} follows a normal distribution with mean $\mu = 300 \times \mu_A = 0$ and variance $\sigma^2 = 300 \times \sigma_A^2 = 1200$. Thus $P(S_{300} < 5) = 0.5557$.

5.33 $P(Y > 1.5) \simeq 0.90$.

5.34

a) $P(\overline{X} > 13) = 0.131$.
b) $P(\min(X_1, X_2, X_3, X_4, X_5) < 10) = 0.5785$. Note that the cumulative distribution function of the minimum is $G(k) = 1 - [1 - \phi(k)]^n$, where $\phi(k)$ is the distribution function of the reduced normal distribution.
c) $P(\min(X_1, X_2, X_3, X_4, X_5) > 15) = 0.292$.

5.35 $n_1 = 2000p(1-p)$, $n_2 = 200p(1-p)$, $0 \leq p \leq 1$.

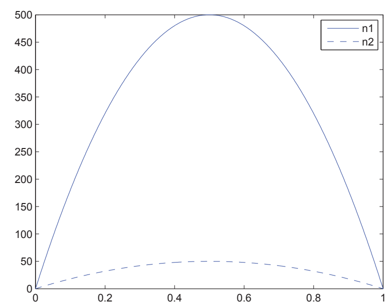

Fig .1: Exercise **5.35**.

5.36 $n \geq 131$.

5.37

a) $P(|f_A - p| < \varepsilon) = 1 - \frac{p(1-p)}{n\varepsilon^2} \Rightarrow n \geq 640$.
b) p is unkown, so we assume $p = q = 0.5$. Thus $n \geq 1000$.

5.38 $n \geq 15840$.

5.39
$n \geq 46875$.

5.40
$n \geq 90000$.

5.41
$n \geq 4000$.

5.42
$\varepsilon = 0.09428$.

5.43

a) $n \geq 18000$.
b) $n \geq 50000$.

5.44

a) $n \geq 46875$.
b) $n \geq 62500$.

5.45 $n \geq 13750$.

5.46 $n \geq 62500$.

5.47 $n \geq 40000$.

5.48

a) $n \geq 510$.
b) $n \geq 1000$.

5.49 $L(X_1,\ldots,X_n;\mu) = \frac{1}{(2\pi)^{n/2}} e^{-\sum_{i=1}^{n}(X_i-\mu)^2}$.

5.50 $\hat{\lambda} = \frac{1}{\bar{X}} = \frac{1}{3.811} \simeq 0.262$.

5.51 $\hat{\mu} = 2.860$, $\hat{\sigma}^2 = 3.050$.

5.52 $\hat{\mu} = 2.614$, $\hat{\sigma}^2 = 3.076$.

5.53 $\hat{\mu} = 3.617$, $\hat{\sigma^2} = 6.077$.

5.54 $\hat{\mu} = 5.02$, $\hat{\sigma}^2 = 0.997$.

5.55

a) $\rho_{XY} = \sqrt{ac}$.
b) $E(X) = \frac{bc+d}{1-ac}$. $E(Y) = \frac{ad+b}{1-ac}$.

5.56

a) $\hat{\alpha} = 4.4380, \hat{\beta} = 1.4568$.
b) $\hat{\sigma^2} = 0.242$.

5.57 $\hat{\alpha} = 1.967$, $\hat{\beta} = 1.078$, $\hat{\sigma^2} = 0.0597$.

5.58

a) $\rho_{XY} = 0.9883$.
b) $\hat{\alpha} = 1.244$, $\hat{\beta} = 1.949$, $\hat{\sigma}^2 = 0.090$.

5.59

a) $\rho_{XY} = 0.9881$.
b) $\hat{\alpha} = 1.197$, $\hat{\beta} = 2.203$, $\hat{\sigma^2} = 0.177$.

5.60

a) $\mu \in [149.876, 150.123]$.
b) $\mu \in [149.837, 150.162]$.

5.61

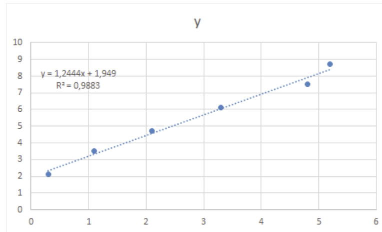

Fig .2: Exercise **5.58**.

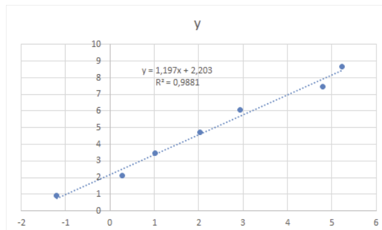

Fig .3: Exercise **5.59**.

a) $\mu \in [585.016, 585.273]$.
b) $P(\bar{X} - \sigma \leq \mu \bar{X} + \sigma) = 0.9544$.
c) $n = 44$.
d) $\mu \in [584.852, 585.437]$.

5.62

a) $\mu \in [99.871, 100.128]$.
b) $\mu \in [99.708, 100.292]$.

6.1

a) $\beta = 0.7298$.

b) Power$=1-\beta = 1-0.7298 = 0.2702$.

c) Increase the size of the sample.

d) The test statistics is $z_0 = \frac{1.25-1.22}{o.5/\sqrt{62}} = 0.47$. The critical value is $z_c = \pm 1.64$, thus the rejection region is $z < -1.64 \cup z > 1.64$. As $-1.64 < z_0 < 1.64$ then there is no reason to reject the null hypothesis, that is, there is no reason to suspect that the population mean is not 1.22.

6.2
The test statistics is $z_0 = \frac{204-207}{5/\sqrt{36}} = -3.6$. The critical value is $z_c = \pm 2.57$. The rejection region is $z < -2.57 \cup z > 2.57$. As $z_0 < -2.57 = z_c$, then there is strong evidence to reject the null hypothesis, thus the mean of the population is not 207.

6.3

a) $H_0: \mu > 21000$
$H_1: \mu \leq 21000$

b) Power$=1-\beta = 0.9525$ (rejection region $z \leq -2.06$).

6.4

a) The sample size increases and α is held constant, then β decreases and the power of the test increases.

b) α is decreased and the sample size is held constant, then β will increase and the power of the test will decrease.

6.5

a) $H_0: \mu = 1\text{h}55\text{m}$
$H_1: \mu \leq 1\text{h}55\text{m}$

The test statistics is $z_0 = \frac{110-115}{10/\sqrt{32}} = -2.83$. The critical value is $z_c = -2.33$ and the rejection region is $z < -2.33$. As $z_0 < z_c$ there is strong evidence to reject the null hypothesis, thus, there is a reason to believe that there is a decrease in the machines' operating time.
Note that $1\text{h}55\text{m} = 115\text{m}$ and $1\text{h}50\text{m} = 110\text{m}$.

b) $\beta = 0.0004$.

6.6 $H_0: \mu = 30\text{m}$
$H_1: \mu \neq 30\text{m}$

The test statistics is $z_0 = \frac{28.5-30}{7/\sqrt{32}} = -1.21$. The critical value is $z_c = \pm 1.96$. The rejection region is $z < -1.96 \cup z > 1.96$. As $-1.96 < z_0 < 1.96$, there is no evidence to reject the null hypothesis, thus we cannot conclude that students are right to be doubtful.

6.7 $H_0: \mu = 230$
$H_1: \mu \neq 230$

The test statistics is $z_0 = \frac{215-230}{45.2/\sqrt{37}} = -2.02$. The critical value is $z_c = \pm 2.06$. The rejection region is $z < -2.06 \cup z > 2.06$. As $-2.06 < z_0 < 2.06$ then there is not enough evidence to reject the null hypothesis, thus we cannot conclude that the mean revenue has changed.

6.8
$H_0: \mu = 37$
$H_1: \mu > 37$

The test statistics is $z_0 = \frac{37.5-37}{9/\sqrt{187}} = 0.76$. The critical value is $z_c = 2.33$. The rejection region is $z > 2.33$. As $z_0 < 2.33$ then there is not enough evidence to reject the null hypothesis, thus we cannot conclude that the makeover has influenced the period of the visits to the swimming club.

6.9 $H_0: \mu = 29.3$
$H_1: \mu < 27.6$

The test statistics is $z_0 = \frac{27.6-29.3}{7.39/\sqrt{41}} = -1.47$. The critical value is $z_c = -1.64$. The rejection region is $z < -1.64$. As $z_0 > -1.64$ then there is not enough evidence to reject the null hypothesis, thus we cannot conclude that the mean lactate level, after two basketball games, is significantly lower than before at $\alpha = 0.05$.

6.10
$H_0: \mu_1 - \mu_2 = 0$
$H_1: \mu_1 - \mu_2 \geq 0$

The test statistics is $z_0 = \frac{5-4.2-0}{\sqrt{\frac{1.3^2}{32}+\frac{0.9^2}{37}}} \sim 2.93$. The critical value is $z_c = 2.06$. The rejection region is $z > 2.06$. As $z_0 > 2.06$ then there is enough evidence to reject the null hypothesis, thus the government claim is accepted, that is a student graduating from university C has taken more math classes than a student graduating from university D.

6.11
$H_0:\ \mu_1-\mu_2=0$
$H_1:\ \mu_1-\mu_2\geq 0$
The test statistics is $z_0=\frac{91.2-87.3-0}{\sqrt{\frac{8.3^2}{35}+\frac{7.8^2}{37}}}\sim 2.05$. The critical value is $z_c=\pm 2.58$. The rejection region is $z<-2.58\cup z>2.58$. As $-2.58<z_0<2.58$ then there is not enough evidence to reject the null hypothesis, thus there is not enough evidence to support that one methodology is more efficient than the other.

6.12
$H_0:\ \mu_1-\mu_2=0$
$H_1:\ \mu_1-\mu_2\leq 0$
The test statistics is $z_0=\frac{79.2-77.5-0}{\sqrt{\frac{11.2^2}{57}+\frac{5.9^2}{57}}}\sim 1.01$. The critical value is $z_c=-2.33$. The rejection region is $z<-2.33$. As $z_0>-2.33$ then there is not enough evidence to reject the null hypothesis, thus there is not enough evidence to conclude that the students coming from a phylosophy background have average score greater than the students coming from a math background.

6.13
$H_0:\ \mu_1-\mu_2=0$
$H_1:\ \mu_1-\mu_2\geq 0$
The test statistics is $z_0=\frac{145-141-0}{\sqrt{\frac{13.2^2}{115}+\frac{9^2}{103}}}\sim 2.66$. The critical value is $z_c=1.64$. The rejection region is $z>1.64$. As $z_0>1.64$ then there is strong evidence to reject the null hypothesis, thus the FootPremier firm was successful.

6.14
$H_0:\ \mu_1-\mu_2=0$
$H_1:\ \mu_1-\mu_2\geq 0$
The test statistics is $z_0=\frac{2.3-2.0-0}{\sqrt{\frac{0.6^2}{43}+\frac{0.75^2}{44}}}\sim 2.06$. The critical value is $z_c=1.76$. The rejection region is $z>1.76$. As $z_0>1.76$ then there is strong evidence to reject the null hypothesis, thus the vitamin supplement accelerates the weight loss of adults in 2 weeks.

6.15
$H_0:\ \mu_1-\mu_2=0$
$H_1:\ \mu_1-\mu_2\geq 0$
The test statistics is $z_0=\frac{97-68.5-0}{\sqrt{\frac{2.7^2}{43}+\frac{1.3^2}{32}}}\sim 60.44$. The critical value is $z_c=1.28$. The rejection region is $z>1.28$. As $z_0>1.28$ then there is strong evidence to reject the null hypothesis, that is the mean alkalinity of water in lower reaches of this river is greater than in the upper reaches.

6.16
$H_0:\ \mu_1-\mu_2=0$
$H_1:\ \mu_1-\mu_2\leq 0$
The test statistics is $z_0=\frac{79-83-0}{\sqrt{\frac{7^2}{67}+\frac{9^2}{52}}}\sim -2.64$. The critical value is $z_c=-1.76$. The rejection region is $z<-1.76$. As $z_0<-1.76$ then there is strong evidence to reject the null hypothesis, that is the boys did better than the girls at a significance level of $\alpha=0.04$.

6.17
$H_0:\ \mu_1-\mu_2=0$
$H_1:\ \mu_1-\mu_2\neq 0$
The test statistics is $z_0=\frac{197-200-0}{\sqrt{\frac{5^2}{57}+\frac{5^2}{43}}}\sim -2.97$. The critical value is $z_c=\pm 2.57$. The rejection region is $z<-2.57\cup z>2.57$. As $z_0<-2.57$ then there is strong evidence to reject the null hypothesis, that is the mean breaking strength of the copper wire from the manufacturer A differs from that supplied by manufacturer B.

6.18
$H_0: \ p = 0.5$
$H_1: \ p \neq 0.5$
The value of $\hat{p}$ is $\hat{p} = \frac{22}{45} \sim 0.49$. The test statistics is $z_0 = \frac{0.49-0.5}{\sqrt{\frac{0.49 \cdot 0.51}{45}}} \sim -0.13$. The critical value is $z_c = \pm 1.96$. The rejection region is $z < -1.96 \cup z > 1.96$. As $-1.96 < z_0 < 1.96$ then there is not enough evidence to reject the null hypothesis, thus Hanna's believe cannot be contradicted.

6.19 $H_0: \ p = 0.25$
$H_1: \ p < 0.25$
The value of $\hat{p}$ is $\hat{p} = \frac{13}{57} \sim 0.23$. The test statistics is $z_0 = \frac{0.23-0.25}{\sqrt{\frac{0.25 \cdot 0.75}{57}}} \sim -0.35$. The critical value is $z_c = -2.33$. The rejection region is $z < -2.33$. As $z_0 > -2.33$ then there is no evidence to reject the null hypothesis so the civil rights group cannot be accepted.

6.20
$H_0: \ p = 0.25$
$H_1: \ p \neq 0.25$
The value of $\hat{p}$ is $\hat{p} = \frac{10}{53} \sim 0.19$. The test statistics is $z_0 = \frac{0.19-0.25}{\sqrt{\frac{0.25 \cdot 0.75}{53}}} \sim -1.01$. The critical value is $z_c = \pm 1.64$. The rejection region is $z < -1.64 \cup z > 1.64$. As $-1.64 < z_0 < 1.64$ then there is not strong evidence to reject the null hypothesis, so there no reason to suspect of any change in the proportion of people, aged between 20 and 34, with an IQ of over 120.

6.21
$H_0: \ p = 0.08$
$H_1: \ p > 0.08$
The test statistics is $z_0 = -4$, so it does not belong to the rejection region (to the right on the normal graph), so there is not strong evidence to reject the null hypothesis, so the competitor's claim could not be accepted.

6.22
$H_0: \ p = 0.2$
$H_1: \ p < 0.2$
The value of $\hat{p}$ is $\hat{p} = \frac{13}{35} \sim 0.37$. The test statistics is $z_0 = \frac{0.37-0.2}{\sqrt{\frac{0.2 \cdot 0.8}{35}}} \sim 2.51$. The critical value is $z_c = -2.06$. The rejection region is $z < -2.06$. As $z_0 > -2.06$ then there is not strong evidence to reject the null hypothesis, so the manager's claim wasn't verified.

6.23
$H_0: \ p = 0.1$
$H_1: \ p > 0.1$
The value of $\hat{p}$ is $\hat{p} = \frac{17}{45} \sim 0.38$. The test statistics is $z_0 = \frac{0.38-0.1}{\sqrt{\frac{0.1 \cdot 0.9}{45}}} \sim 6.26$. The critical value is $z_c = 1.29$. The rejection region is $z > 1.29$. As $z_0 > 1.29$ then there is strong evidence to reject the null hypothesis, so the club won't increase the prices of tickets.

6.24
$H_0: \ p = 0.63$
$H_1: \ p > 0.63$
The value of $\hat{p}$ is $\hat{p} = \frac{26}{35} \sim 0.74$. The test statistics is $z_0 = \frac{0.74-0.63}{\sqrt{\frac{0.63 \cdot 0.37}{35}}} \sim 1.35$. The critical value is $z_c = 1.64$. The rejection region is $z > 1.64$. As $z_0 < 1.29$ then there is not enough evidence to reject the null hypothesis, so it is not clear that the new drug is more efficient, at a level of significance of $\alpha = 0.05$.

6.25
$H_0: \ p = 0.04$
$H_1: \ p < 0.04$
The value of $\hat{p}$ is $\hat{p} = \frac{17}{156} \sim 0.06$. The test statistics is $z_0 = \frac{0.06-0.04}{\sqrt{\frac{0.04 \cdot 0.96}{156}}} \sim 1.27$. The critical value is $z_c = -1.75$. The rejection region is $z < -1.75$. As $z_0 > -1.75$ then the null hypothesis cannot be rejected, thus we cannot say that the machine has improved its efficiency.

6.26
$H_0: \ p_1 - p_2 = 0$
$H_1: \ p_1 - p_2 \neq 0$
The values of $\hat{p}_1$ and $\hat{p}_2$ are $\hat{p}_1 = \frac{17}{53} \sim 0.32$ and $\hat{p}_2 = \frac{11}{40} \sim 0.28$.
The value of $p^\star$ is $p^\star = \frac{17+11}{53+40} \sim 0.30$.
The test statistics is $z_0 = \frac{0.32-0.28-0}{\sqrt{0.3 \cdot 0.7\left(\frac{1}{53}+\frac{1}{40}\right)}} \sim 0.42$. The critical value is $z_c = \pm 1.96$. The rejection region is $z < -1.96 \cup z > 1.96$. As $z_0 < 1.96$ then there isn't enough evidence to reject the null hypothesis, so the proportions of orange candies, in the plain and almond varieties, do not differ at a level $\alpha = 0.05$.

6.27
$H_0: \ p_1 - p_2 = 0$
$H_1: \ p_1 - p_2 \neq 0$
The values of $\hat{p}_1$ and $\hat{p}_2$ are $\hat{p}_1 = \frac{62}{75} \sim 0.83$ and $\hat{p}_2 = \frac{37}{60} \sim 0.62$.
The value of $p^\star$ is $p^\star = \frac{62+37}{75+60} \sim 0.73$.
The test statistics is $z_0 = \frac{0.83-0.62-0}{\sqrt{0.73 \cdot 0.27\left(\frac{1}{75}+\frac{1}{60}\right)}} \sim 2.73$. The critical value is $z_c = \pm 2.57$. The rejection region is $z < -2.57 \cup z > 2.57$. As $z_0 > 2.57$ then there is strong evidence to reject the null hypothesis, so men and women differ in what concerns political choices of this candidate at a level $\alpha = 0.01$.

6.28
$H_0: \ p_1 - p_2 = 0$
$H_1: \ p_1 - p_2 < 0$
The values of $\hat{p}_1$ and $\hat{p}_2$ are $\hat{p}_1 = \frac{15}{160} \sim 0.09$ and $\hat{p}_2 = \frac{18}{175} \sim 0.10$.
The value of $p^\star$ is $p^\star = \frac{15+18}{160+175} \sim 0.099$.
The test statistics is $z_0 = \frac{0.09-0.10-0}{\sqrt{0.099 \cdot 0.901\left(\frac{1}{175}+\frac{1}{160}\right)}} \sim -0.31$. The critical value is $z_c = -2.33$. The rejection region is $z < -2.33$. As $z_0 > -2.33$ then there is no reason to reject the null hypothesis, so med A is not less effective than med B, at a level $\alpha = 0.01$.

6.29 $H_0: \ p_1 - p_2 = 0$
$H_1: \ p_1 - p_2 < 0$
The values of $\hat{p}_1$ and $\hat{p}_2$ are $\hat{p}_1 = \frac{87}{190} \sim 0.46$ and $\hat{p}_2 = \frac{93}{150} \sim 0.62$.
The value of $p^\star$ is $p^\star = \frac{87+93}{190+150} \sim 0.53$.
The test statistics is $z_0 = \frac{0.46-0.62-0}{\sqrt{0.53 \cdot 0.47\left(\frac{1}{190}+\frac{1}{150}\right)}} \sim -2.94$.
The critical value, for $\alpha = 0.05$ is $z_c = -1.64$. The rejection region is $z < -1.64$.
As $z_0 < -1.64$ then there is strong evidence to reject the null hypothesis, so the new drug is more effective at a level $\alpha = 0.05$.
The critical value, for $\alpha = 0.1$ is $z_c = -1.29$. The rejection region is $z < -1.29$. As $z_0 < -1.29$ then there is strong evidence to reject the null hypothesis, so the new drug is more effective at a level $\alpha = 0.1$.

6.30
$H_0: \ p_1 - p_2 = 0$
$H_1: \ p_1 - p_2 \neq 0$

The values of $\hat{p}_1$ and $\hat{p}_2$ are $\hat{p}_1 = \frac{70}{87} \sim 0.80$ and $\hat{p}_2 = \frac{55}{67} \sim 0.82$.
The value of $p^\star$ is $p^\star = \frac{70+55}{87+67} \sim 0.81$.
The test statistics is $z_0 = \frac{0.80-0.82-0}{\sqrt{0.81 \cdot 0.19\left(\frac{1}{87}+\frac{1}{67}\right)}} \sim -0.31$.
The critical value, for $\alpha = 0.05$ is $z_c = \pm 1.96$. The rejection region is $-1.96 > z \cup z > 1.96$. As $-1.96 < z_0 < -1.96$ then there is not enough evidence to reject the null hypothesis, so the audiences of the two channels are not different, in what concerns watching news preferences, at a level $\alpha = 0.05$.

6.31 $H_0: \; p_1 - p_2 = 0$
$H_1: \; p_1 - p_2 < 0$
The values of $\hat{p}_1$ and $\hat{p}_2$ are $\hat{p}_1 = \frac{15}{115} \sim 0.13$ and $\hat{p}_2 = \frac{25}{97} \sim 0.26$.
The value of $p^\star$ is $p^\star = \frac{15+25}{115+97} \sim 0.19$.
The test statistics is $z_0 = \frac{0.13-0.26-0}{\sqrt{0.81 \cdot 0.19\left(\frac{1}{115}+\frac{1}{97}\right)}} \sim -2.40$.
The critical value, for $\alpha = 0.02$ is $z_c = -2.06$. The rejection region is $-2.06 > z$.
As $z_0 < -2.06$ then there is strong evidence to reject the null hypothesis, so the advertising campaign had success, at a level $\alpha = 0.02$.

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Probability and Statistics —Selected Problems
J.A. Tenreiro Machado, Carla M.A. Pinto
2014, 208p., Papercover
ISBN 978-1-62155-006-8